UNIT-1

COMPLEX NUMBERS

CONTENTS

COMPLEX NUMBERS

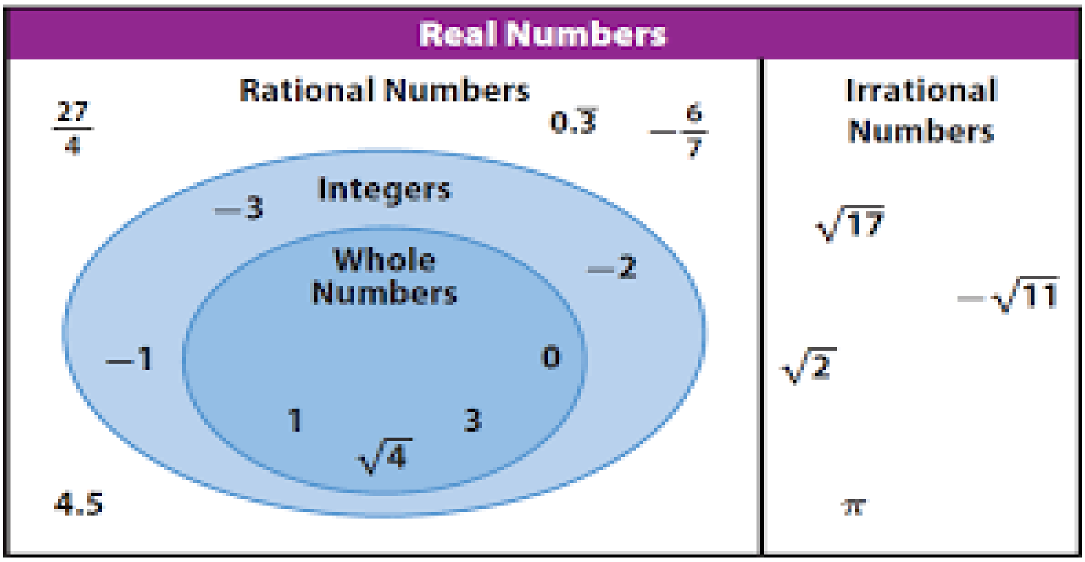
1.1. Introduction:

In this Chapter we briefly discuss about the introduction of complex numbers, algebra of complex numbers, complex conjugate, modulus ,amplitude of complex numbers. This is followed by cube roots of unity and De- Moivre's Theorem to find the nth roots of complex numbers.

After completion of this chapter we will be able to use complex number concept in electricity to find amplitude ,impendence, etc. and imaginary number in signal processing in Radar system and also in biology(Brain waves).

Real Numbers(R):

It is a fundamental fact about real numbers that the square of any real number is never negative.



Thus there is no real $'x'$ which satisfies the equation $x^2 + 1 = 0$ .We shall extend the Real numbers set which include numbers which satisfy such equations.

The equation $x^2 + 1 = 0$

$$\Rightarrow x^2 = -1$$

$$\Rightarrow x = \sqrt{-1}.$$

Imaginary Numbers:

Let us represent $\sqrt{-1}$ as 'i'(imaginary unit)

$$i = \sqrt{-1}$$

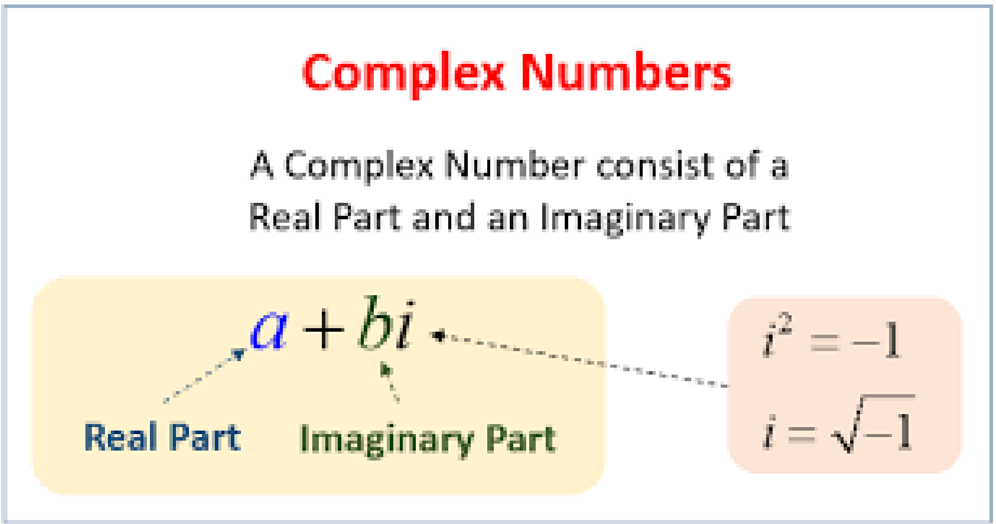
$$i^2 = i.i = \sqrt{-1}.\sqrt{-1} = -1$$

$$i^2 = -1$$

$$2i, -3i, \tfrac{1}{2}i, \sqrt{3}\ i, etc\ are\ imaginary\ numbers.$$

1.2 Complex Numbers:

A number of the form $\boldsymbol{a + ib}$ where $'a'$ and $'b'$ are real numbers and $i = \sqrt{-1}$ is said to be complex number.



Complex Numbers Set

$$C = \{a + ib |\ a, b \in R \ and \ i = \sqrt{-1}\}$$

If $z = a + ib$ is a complex number then

$$Real\ part\ of\ z = Re(z) = a$$

$$Imaginary\ part\ of\ z = Img(z) = b$$

Example 1.1

If $z = 2 + 3i$ then $Re(z) = 2\ and\ Img(z) = 3$

Example 1.2

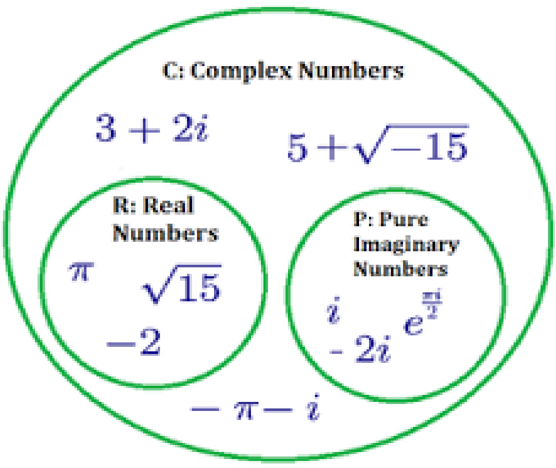
If $z = 2 = 2 + i.0\ then\ Re(z) = 2, Img(z) = 0$

$\{a + i0 | a \in R, i = \sqrt{-1}\}$ **(purely real numbers)**

Example 1.3

$$If\ z = -4i = 0 + (-4)i then\ Re(z) = 0, Img(z) = -4$$

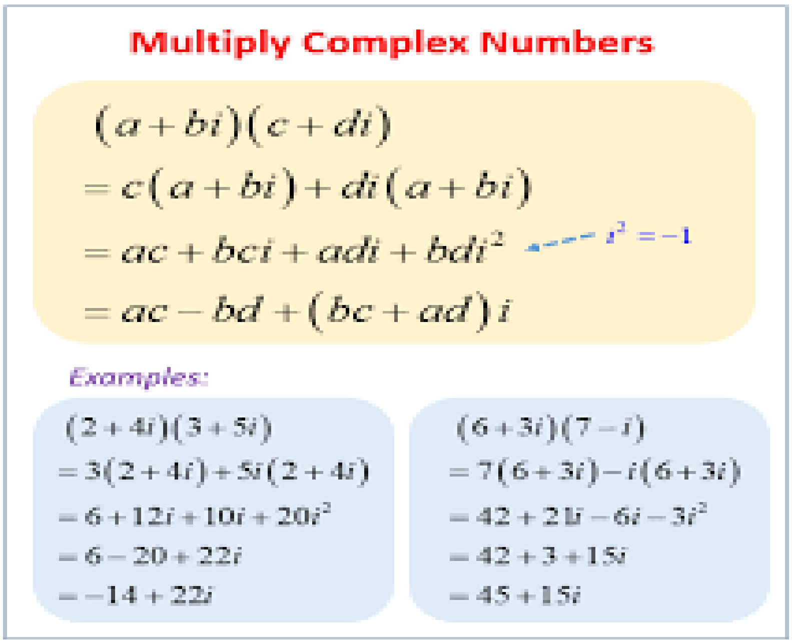
$\{0 + ib | b \in R, i = \sqrt{-1}\}$ **(purely imaginary numbers)**



Algebra of Complex Number s

If $z_1 = a + ib\ , z_2 = c + id$

1) Addition: $z_1 + z_2 = (a + c) + i(b + d)$
2) Substraction: $z_1 - z_2 = (a - c) + i(b - d)$
3) Multiplication: $z_1 . z_2 = (ac - bd) + i(ad + bc)$



Properties of Addition:

It is clear from definitions that

$$z = x + iy \, , z_1 = a + ib \, , z_2 = c + id \, , 0 = 0 + i0 , 1 = 1 + i0$$

(i) Closure property: $z_1 + z_2 \in C$
(ii) Commutative property: $z_1 + z_2 = z_2 + z_1$
(iii) Additative Identity: $z + 0 = 0$
(iv) Additative inverse: $z + (-z) = 0 \; where \; -z = -x - iy$

Properties of Multiplication:

(i) Closure property: $z_1 . z_2 \in C$
(ii) Commutative property: $z_1 z_2 = z_2 z_1$
(iii) Associative property: $(z_1 z_2) z_3 = z_1 (z_2 z_3)$
(iv) Multiplicative Identity: $z . 1 = z$
(v) Multiplicative inverse: $z z^{-1} = 1 \;\; if \; z \neq 0$

Complex Conjugate: $(\bar{z})$

If $z = a + ib$ then $\bar{z} = a - ib$

Given a complex number	Its conjugate
$a + bi$	$a - bi$
$a - bi$	$a + bi$

=

Example1.4

If $z = 2 + 3i$ the $\bar{z} = 2 - 3i$

Example1.5

If $z = -4i$ $then$ $\bar{z} = 4i$

Modulus $(|z|)$:

If $z = a + ib$ $then$ $modulus$ of z is $|z| = \sqrt{a^2 + b^2}$

Example1.6

If $z = 2 + 3i$ $then$ $|z| = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13}$

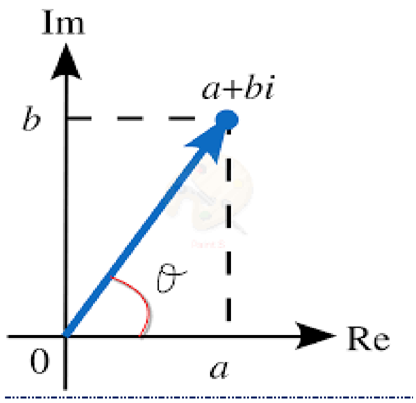
Example 1.7

If $z = 3i$ $then$ $|z| = \sqrt{0^2 + 3^2} = 3$

Example 1.8

If $z = 5$ $then$ $|z| = \sqrt{5^2 + 0^2}$=5

Argument (argz): If $z = a + ib$ $then$ $argument$ of z $= argz = \tan^{-1}\frac{b}{a}$



Example1.9

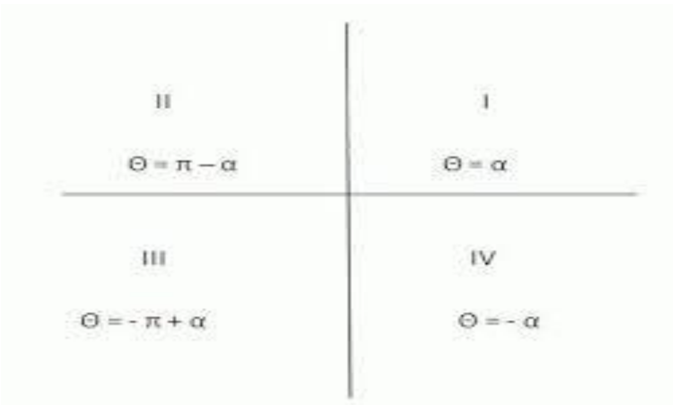
If $z = 2 + 3i$ $then\ argz = \tan^{-1}\frac{3}{2}$

Method to find argument of z=x+iy

Step 1: Find $\alpha = \tan^{-1}\left|\frac{y}{x}\right|$ and this gives the value of α in the 1st quadrant.

Step 2:Find the quadrant in which z lies with the help of sign of x and y co-ordinates.

Step 3: Then $argz = \theta$ will be $\alpha, \pi - \alpha, \alpha - \pi\ and\ (-\alpha)$ according as z lies in the first,second,third and fourth quadrant.



Example 1.10

Find the argument of $z = -1 + i\sqrt{3}$?

Ans- $z = -1 + i\sqrt{3} \rightarrow (-1, \sqrt{3})$

Since this is a point in the second quadrant so argz=$argz = \theta = \pi - \alpha$

Where $\alpha = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right|$

$\Rightarrow \alpha = \tan^{-1}\sqrt{3}$

$\Rightarrow tan\alpha = \sqrt{3}$

$\Rightarrow tan\alpha = tan\frac{\pi}{3}$

$\Rightarrow \alpha = \frac{\pi}{3}$

$\therefore\ \theta = argz = \pi - \alpha$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ (Ans)

Multiplicative Inverse :

If $z = a + ib, z \neq 0$ $then\ multiplicative\ inverse\ of\ z = z^{-1} = \frac{1}{z} = \frac{1}{a+ib}$

$= \frac{a-ib}{(a+ib)(a-ib)}$

$= \frac{a-ib}{a^2-i^2b^2}$

$= \frac{a-ib}{a^2+b^2} = \frac{\bar{z}}{|z|}$

$$z^{-1} = \frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2} = \frac{\bar{z}}{|z|}$$

1.3 Geometrical Representation of a Complex Numbers:

The complex number $z = a + ib$ is represented by the ordered pair (a, b)

$z = a + ib \rightarrow (a, b)$

$$|z| = r = \sqrt{a^2 + b^2}$$

$$argz = \theta = \tan^{-1}\frac{b}{a}$$

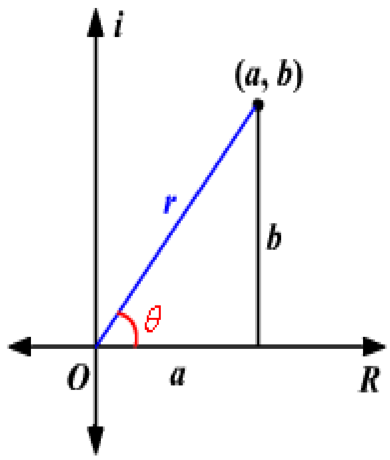
$$\bar{z} = a - ib \rightarrow (a, -b)$$

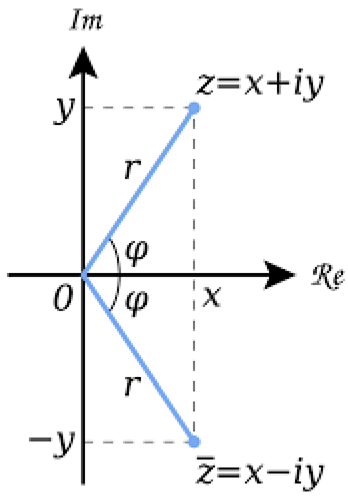
X-axis → Real axis (real part complex number is represented by X-axis).

Y-axis → Imaginary axis (imaginary part is represented by Y-axis)

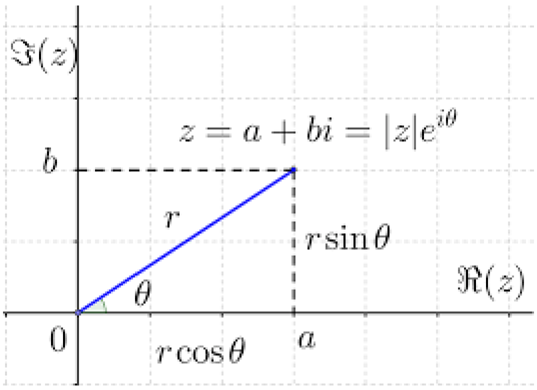
Cartesian plane → Complex plane (There exist a one to one correspondence between the elements complex set and point on this plane)

Complex plane→ Argand plane (As the identification of a complex number on aplane was proposed by Jean –Robert Argand**)**





Polar Representation:



$$z = a + ib,$$

$$|z| = \sqrt{a^2 + b^2}$$

$$\theta = argz$$

In the triangle $\sin\theta = \frac{b}{r}$

$$\Rightarrow b = r\sin\theta$$

$$\cos\theta = \frac{a}{r}$$

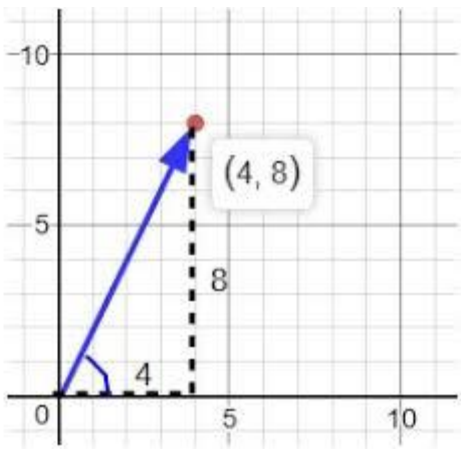
$$a = r\cos\theta$$

Polar form

$$a + ib = r\cos\theta + i\,r\sin\theta = r(\cos\theta + i\,sin\theta)$$

Example :1.11

Represent $z = 4 + i8$ diagrametically in the complex plane.



Example:1.12

Express $z = 1$ in polar form.

Ans: $z = 1$,

$$|z| = \sqrt{1^2 + 0^2} = 1 = r$$

$$\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{0}{1} = \tan^{-1}0 = 0$$

$$1 = 1.\cos 0 + i1.\sin 0 = \cos 0 + i\sin 0$$

As $\cos(2k\pi + \theta) = \cos\theta$

$$\sin(2k\pi + \theta) = \sin\theta$$

$$1 = \cos(2k\pi + 0) + i\sin(2k\pi + 0)$$

$$1 = \cos(2k\pi) + i\sin(2k\pi) \qquad k = 0,1,2, \dots\dots \text{(Ans)}$$

Example 1.13

Express $z = 1 + i$ in polar form.

Ans: $z = 1 + i$

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2},$$

$$\theta = \tan^{-1}\frac{1}{1} = \tan^{-1} 1 = \frac{\pi}{4}$$

$$z = 1 + i = r\cos\theta + i\, r\sin\theta$$

$$= \sqrt{2}\,\cos\frac{\pi}{4} + i\sqrt{2}\sin\frac{\pi}{4}$$

$$= \sqrt{2}\left(\cos(2k\pi + \tfrac{\pi}{4}\right) + i\sin(2k\pi + \tfrac{\pi}{4})) \qquad \text{(Ans)}$$

1.4 Properties:

If z_1, z_2 are two complex numbers then

(i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

(ii) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

(iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$, $z_2 \neq 0$

(iv) $|z_1 . z_2| = |z_1||z_2|$

(v) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

(vi) $\arg(z_1 z_2) = arg z_1 + arg z_2$

(vii) $arg\left(\frac{z_1}{z_2}\right) = arg z_1 - arg z_2$

PROBLEMS 1(A)

Problem 1.1

Express the $\frac{3+5i}{2-3i}$ in $a+ib$ form.

Ans: $\frac{3+5i}{2-3i}=\frac{(3+5i)(2+3i)}{(2-3i)(2+3i)}$

$$=\frac{6-15+i(10+9)}{4-i^2 3^2}$$

$$=\frac{-9+i19}{4+9}$$

$$=\frac{-9}{13}+i\frac{19}{13} \quad \text{(Ans)}$$

Assignment 1.1

Express the following in $a+ib$ form.

1) $\frac{2+3i}{5-2i}$

2) $\frac{(1+i)^2}{3-i}$

3) $\frac{1+i}{1-i}$

Problem1.2

Find the conjugate of $\frac{1-i}{3+i}$.

Ans: $z=\frac{1-i}{3+i}=\frac{(1-i)(3-i)}{(3+i)(3-i)}$

$$=\frac{3-1+i(-1-3)}{3^2-i^2}=\frac{2-4i}{9+1}=\frac{2-4i}{10}=\frac{2}{10}-i\frac{4}{10}=\frac{1}{5}-i\frac{2}{5}$$

$\bar{z}=\frac{1}{5}+i\frac{2}{5}$ (Ans)

Assignment 1.2

Find the conjugate of the following complex numbers

1) $\frac{3+5i}{2-3i}$
2) $\frac{1+i}{1-i}$
3) $\frac{2+3i}{5-2i}$

.

Problem1.3

Find the multiplicative inverse of $3+5i$

Ans: $(3+5i)^{-1} = \frac{1}{3+5i} = \frac{(3-5i)}{(3+5i)(3-5i)}$

$= \frac{3-5i}{3^2-(5i)^2}$

$= \frac{3-5i}{9-(-25)} = \frac{3-5i}{34} = \frac{3}{24} - i\frac{5}{24}$ (Ans)

Assignment 1.3

Find the multiplicative inverse of

1) $3+2i$ (2) $3i$ (3) $1-i$

Problem 1.4

Find the modulus of $(1+i)^2$.

Ans: $z = (1+i)^2 = 1-1+2i = 0+2i$.

$\therefore |z| = \sqrt{0^2+2^2} = \sqrt{4} = 2$ (Ans)

Assignment 1.4.

Find the modulus of the followings.

1) $\frac{3+4i}{3-2i}$ 2) $4i$ 3) $\frac{1-i}{1+2i}$

Problem 1.5

Find the argument of $z = 1 + i$?

Ans- $z = 1 + i \rightarrow (1,1)$

Since this is a point in the 1st quadrant so argz=$argz = \theta = \alpha$

Where $\alpha = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{1}{1}\right|$

$$\Rightarrow \alpha = \tan^{-1} 1$$

$$\Rightarrow tan\alpha = 1$$

$$\Rightarrow tan\alpha = tan\frac{\pi}{4}$$

$$\Rightarrow \alpha = \frac{\pi}{4}$$

$$\therefore\ \theta = argz = \alpha$$

$$= \frac{\pi}{4} \quad \text{(Ans)}$$

Assignment 1.5

Find the argument of the followings.

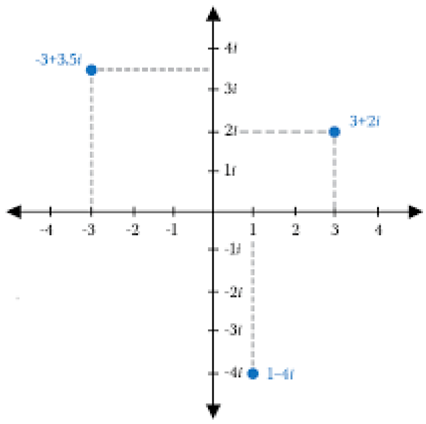
1)$\sqrt{3} - i$ 2) $-1 + i$ 3) $-1 - i$ 4) $1 - i$

Problem1.6

Represent the complex number $3 + 2i, -3 + i3.5, 1 - 4i$ in the complex plane.

Ans: The given complex numbers are represented by the following points

$3 + 2i \rightarrow (3,2), -3 + i3.5 \rightarrow (-3,3.5), 1 - 4i \rightarrow (1,-4)$



Assignment 1.6

Represent the following complex numbers geometrically in the Argand plane

(i) $-1 + i6$ $(ii) - 2 - 4i$ $(iii) 1 - i$ $(iv)\ 3i$

Problems .1.7

$$i = \sqrt{= -1},$$

$$i^2 = -1$$

$$i^3 = i^2 i = (-1).i = -i$$

$$i^4 = i^3.i = (-i).i = -i^2 = -(-1) = 1$$

$$i^{4n} = (i^4)^n = 1^n = 1\ , n \in Z$$

$$i^{4n+1} = i$$

$$i^{4n+2} = -1$$

$$i^{4n+3} = -i$$

Find the value of i^{1011}

Ans: $i^{1011} = i^{4\times252+3} = i^{4\times252} i^3 = i^3 = -i$ (Ans)

Assignment 1.7

Find the value of

$(i)(-i)^{4n+3}$ where n is a positive integer. (ii) $(-i)^{101}$ $(iii)\ i^{45}$

1.5 Cube roots of unity: $(1)^{1/3}$

$$\text{Let } x = \sqrt[3]{1}$$

$$\Rightarrow x^3 = 1$$

$$\Rightarrow x^3 - 1 = 0$$

$$\Rightarrow (x-1)(x^2+x+1) = 0$$

$$\Rightarrow x - 1 = 0, (x^2+x+1) = 0$$

$$\Rightarrow x = 1 (real\ root) \quad x^2 + x + 1 = 0\ (quadratic\ equation)$$

$\therefore x = 1,\ x = \alpha = \frac{-1+i\sqrt{3}}{2}, x = \beta = \frac{-1-i\sqrt{3}}{2},$ are the cube roots of unity.

Properties:

(i) $\bar{\alpha} = \beta, \bar{\beta} = \alpha$

In otherwords ,the two complex roots are conjugates of each other.

(ii) $\alpha^2 = \beta\ , \beta^2 = \alpha$

In otherwords the square of any complex root is the other complex

(iii) $If\ \ \omega = \frac{-1+i\sqrt{3}}{2}\ then\ \ \omega^2 = \frac{-1-i\sqrt{3}}{2}$.

So cube roots of unity are $1,\ \omega,\ \omega^2$

(iv) Since ω is the root of the equation $x^3 = 1$

$$\Rightarrow \omega^3 = 1$$

$$\omega^{3n} = (\omega^3)^n = 1$$

$$\omega^{3n+1} = \omega$$

$$\omega^{3n+2} = \omega^2$$

(vi) Since ω is the complex root of $x^3 - 1 = 0$.
i.e. ω is the root of $x^2 + x + 1 = 0$

So $\omega^2 + \omega + 1 = 0$

Example 1.12.

Prove that $(1+\omega^2)^4 = \omega$ where 1, ω, ω^2 are the cube roots of unity.

Ans. L.H.S.= $(1+\omega^2)^4$

$= (-\omega)^4$ $\quad (As\ \omega^2 + \omega + 1 = 0)$

$= \omega^4$

$= \omega^3\omega$

$= \omega$ $\quad (As\ \omega^3 = 1)$

=R.H.S (Proved)

1.6

De-Moivres Theorem (for integral index)

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta \ \ for\ any\ integer.$$

De –Moivres Theorem (for the rational Index)

$$(\cos\theta + i\sin\theta)^{\frac{p}{q}} = \cos(\tfrac{p}{q}\theta) + \sin(\tfrac{p}{q}\theta)$$

where $\frac{p}{q}$ a rational number with $q > 0$.

Application:

Example 1.12

Find the cube roots of unity.

Ans : $x = 1^{\frac{1}{3}}$

$$\Rightarrow x = (\cos 2\pi k + i\sin 2\pi k)^{\frac{1}{3}}$$

$$= \cos\frac{2\pi k}{3} + i\sin\frac{2\pi k}{3} \qquad k = 0,1,2$$

$$k = 0 : x = \cos 0 + i \sin 0 = 1 + i0 = 1$$

$$k = 1 : x = \cos\tfrac{2\pi}{3} + i \sin\tfrac{2\pi}{3} = -\tfrac{1}{2} + i\tfrac{\sqrt{3}}{2} \qquad (As\ \tfrac{2\pi}{3}=120\ )$$

$$k = 2 : x = \cos\tfrac{2\pi\times 2}{3} + i \sin\tfrac{2\pi\times 2}{3}$$

$$= \left(\cos\tfrac{4\pi}{3} + i \sin\tfrac{4\pi}{3}\right) = -\tfrac{1}{2} - i\tfrac{\sqrt{3}}{2} \qquad \text{(Ans)}$$

$$\left(\text{As}\ \ sin\frac{4\pi}{3} = \sin\left(\pi + \frac{\pi}{3}\right) = -sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}\right)$$

$$If \quad a = r(\cos\theta + \mathrm{i}\sin\theta)\ then$$

$$z = r^{\frac{1}{n}}\left\{\cos\frac{2k\pi + \theta}{n} + i \sin\frac{2k\pi + \theta}{n}\right\} \quad k = 0,1,2,3, \ldots \ldots \ldots n-1$$

PROBLEM 1 (B)

Problem 1.1

If $1,\ \omega\ ,\ \omega^2$ are the cube roots of unity prove that

$$(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5 = 32$$

Proof: Since $1,\ \omega\ ,\ \omega^2$ are the cube roots of unity ,so $\omega^3 = 1$

$$and\ 1 + \omega + \omega^2 = 0$$

$$1 + \omega = -\omega^2$$

$$1 + \omega^2 = -\omega$$

$$\omega + \omega^2 = -1$$

L.H.S.$=(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$

$$= (1 + \omega^2 - \omega)^5 + (1 + \omega - \omega^2)^5$$

$$= (-\omega - \omega)^5 + (-\omega^2 - \omega^2)^5 \qquad (1 + \omega = -\omega^2, 1 + \omega^2 = -\omega$$

$$= (-2\omega)^5 + (-2\omega^2)^5$$

$$= (-2)^5\omega^5 + (-2)^5\omega^{10}$$

$= (-2)^5(\omega^5 + \omega^{10})$

$= (-2)^5(\omega^2 + \omega)$ $\quad (\omega^{3n} = 1)$

$= (-2)^5(-1)$ $\quad (\omega + \omega^2 = -1)$

$= (-32).(-1)$

$= 32 = R.H.S$ $\quad$ (Proved)

Assignment1.1

If $1, \omega, \omega^2$ are the cube roots of unity prove that

(i) $(1 - \omega + \omega^2)(1 + \omega - \omega^2) = 4$

(ii) $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^5) = 9$

(iii) $(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^2)\ldots to\ 2n\ factors = 2^{2n}$

(iv) $Evaluate \begin{vmatrix} \omega^6 & \omega^4 \\ -\omega^6 & \omega^5 \end{vmatrix}$.

(v) Find the value of $(1 - \omega + \omega^2)(1 + \omega + \omega^2)$.

Problem 1.2

If $x + \frac{1}{x} = 2\cos\theta$ then show that $x^n + \frac{1}{x^n} = 2\cos n\theta$.

Ans: Given $x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow \frac{x^2+1}{x} = 2cos\theta$

$\Rightarrow x^2 + 1 = 2xcos\theta$

$\Rightarrow x^2 + cos^2\theta + sin^2\theta = 2xcos\theta$

$\Rightarrow x^2 + cos^2\theta - 2xcos\theta = -sin^2\theta$

$\Rightarrow (x - cos\theta)^2 = i^2 sin^2\theta$

$\Rightarrow x - cos\theta = \pm i\ sin\theta$

$\Rightarrow x = cos\theta \pm i\ sin\theta$

$let\ \ x = cos\theta + i\ sin\theta$

$x^n = (cos\theta +\ i\ sin\theta)^n$

$= cosn\theta + isinn\theta \qquad (De - Moivres\ Theorem) \ldots\ (1)$

$$\frac{1}{\mathrm{x^n}} = (\cos\theta + \mathrm{i}\sin\theta)^{-\mathrm{n}} = \cos(-n\theta) + isin(-n\theta)$$

$$= \cos n\theta - i\sin n\theta\ \ (De - Moivres\ Theorem) \ldots\ (2)$$

From equation (1) and equation(2)

$x^n + \frac{1}{\mathrm{x^n}} = 2cosn\theta$ (proved)

Assignment1.2

(i) If $x + \frac{1}{x} = 2\cos\theta$ then show that $x^n - \frac{1}{x^n} = 2\,i\sin n\theta$.

(ii) Find the $1^{\frac{1}{4}}$.

(ii) Find the cube roots of $1 + i$.

Problem 1.3

Find the square roots of $3 + 4i$.

Ans: Let $x\,, y\ \in R$ such that

$$x + iy = \sqrt{3 + 4i}$$

Squaring both sides

$$(x + iy)^2 = 3 + 4i$$

$$\Rightarrow x^2 + (iy)^2 + 2.x.iy = 3 + 4i$$

$$\Rightarrow x^2 - y^2 + i.2xy = 3 + 4i \qquad (As\ \ i^2 = -1)$$

Equating the real and imaginary parts

$$x^2 - y^2 = 3 \ldots\ldots\ldots\ldots(1)$$

$$2xy = 4 \ldots\ldots\ldots(2)$$

$$(a+b)^2 = (a-b)^2 + 4ab$$

So $(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$$\Rightarrow (x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$$

$$Using\ x^2 - y^2 = 3,\ \ 2xy = 4\ \ we\ have$$

$$(x^2+y^2)^2 = (3)^2 + (4)^2 = 9 + 16$$

$$\Rightarrow (x^2+y^2)^2 = 25$$

But since $x^2 + y^2$ is non-negative , so we have

$$\Rightarrow x^2 + y^2 = 5 \ldots\ldots\ldots\ldots\ldots(3)$$

From equation (1) and equation (3)

$$x^2 - y^2 = 3$$

$$\underline{x^2 + y^2 = 5}$$

$$\Rightarrow 2x^2 = 8$$

$$\Rightarrow x = \pm 2$$

$$When\ \ x = \pm 2\ \ then\ \ \ y = \pm 4$$

But $2xy = 4$

As the product of x and y is positive,

$\Rightarrow$ $x\ and\ y\ have\ \ same\ sign.$

Thus if $x = 2 \ \Rightarrow y = 1$, $so\ x + iy = 2 + i.1$

If $x = -2 \Rightarrow y = -1$, $so\ x + iy = -2 + i(-1) = -2 - i$

$\therefore \sqrt{3 + 4i} = \pm(2 + i)$ (Ans)

Assignment 1.3

Find the square roots of

(i) $-15 - 8i$ (ii)$-7 + 24\sqrt{-1}$ $(iii) - 5 + 12i$ $(iv) - 8 + \sqrt{-1}$