Lectures notes

On

STRENGTH OF MATERIALS

Course Code-TH2

Prepared by

Mrs.Shradha Suman Adabar

Department of Mechanical Engineering

(CH-1) (SIMPLE STRESS & STRAIN)

STRESS

* Stress can be defined as the force applied per unit area.
* It is denoted by the symbol 'σ' (sigma)
* Stress can be calculated by

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$\Rightarrow \sigma = \frac{F}{A}$$

* The unit for stress is N/m^2

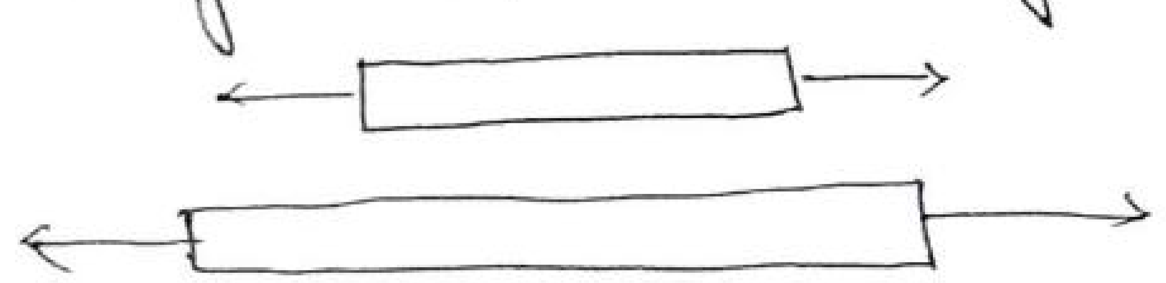
TYPES OF STRESSES :-

Stress can be classified into 2 types. They are

(1) Tensile stress
(2) Compressive stress

(1) TENSILE STRESS

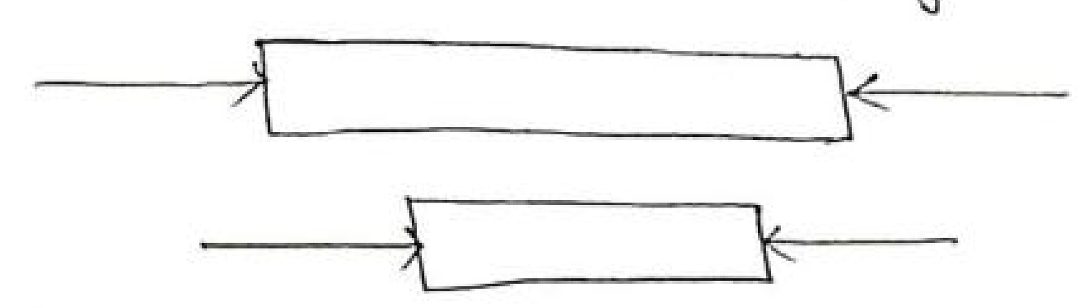
* It can be defined as the stress which is subjected to a body and it results in increase in length of the body.



* It is denoted by the term 'σ_t' (sigma t) (+ve sign)

(2) COMPRESSIVE STRESS :-

* It can be defined as the stress which when applied to a body results in decrease in length of the body.
* It is denoted as 'σ_c' (sigma c) (−ve sign)



STRAIN

* Strain can be defined as the ratio of change in length to the original length.
* $$\text{strain} = \frac{\text{Change in length}}{\text{original length}}$$
* Strain is denoted as ε (Epsilon)

$$\boxed{\varepsilon = \frac{\Delta l}{l}}$$

* $$\boxed{\text{strain} = \frac{\text{Final length} - \text{original length}}{\text{original length}}}$$
* Strain is a unitless quantity.

$$\varepsilon = \frac{\Delta l}{l} = \frac{\cancel{m}}{\cancel{m}}$$

* Strain is a unitless and dimensionless quantity.

ELASTIC LIMIT:-

Elastic limit can be defined as the limit within which a material tends to be elastic. Once a material crosses the elastic limit it becomes plastic.

ELASTIC MATERIAL:-

Material which can return to it's original shape and size after the removal of the deforming force are called elastic materials.

PLASTIC MATERIAL:-

Material which can't return to it's original shape and size after the removal of the deforming force are called as plastic limit.

HOOKE'S LAW:-

It states that "within elastic limit, stress is directly proportional to strain".

$$\sigma \propto \epsilon$$

DEFORMATION OF A BODY UNDER STRESS:-

We know that, according hooke's law

$$\sigma \propto \epsilon$$

$$\sigma = E\epsilon$$

E = Proportionality constant / Young's Modulus.

YOUNG'S MODULUS (E) IMP

we know that $\sigma = E\epsilon$

$$E = \frac{\sigma}{\epsilon}$$

$\Rightarrow$ $\boxed{\text{Young's modulus} = \frac{\text{stress}}{\text{strain}}}$ $\frac{N/m^2}{-}$

* Unit of E is also N/m^2

$\Rightarrow$ Young's modulus can be defined as ratio of the stress to strain.

ELONGATION OF A BODY DUE TO STRESS

i) σ_t $l\uparrow$, $\Delta L = +ve$

ii) σ_c $l\downarrow$, $\Delta L = -ve$

We know that

$$E = \frac{\sigma}{\epsilon} \qquad \left(\begin{array}{l}\sigma = \frac{F}{A} \\ \epsilon = \frac{\Delta L}{L}\end{array}\right)$$

$$\Rightarrow E = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$\Rightarrow E = \frac{F}{A} \times \frac{L}{\Delta L}$$

$$\Rightarrow \Delta L = \frac{F \times L}{A \times E}$$

$\Rightarrow$ $\boxed{\Delta L = \frac{FL}{AE}}$ (Elongation of a body)

(change in length)

Q. A steel rod 1 m long and ~~20 mm~~ 20 X 20 mm in cross-section is subjected to a tensile force of 40 KN. Calculate the elongation of the rod if young's modulus is given 200 GPa

Data given:- steel rod $l = 1\,m = 1000\,mm$

$A = 20 \times 20\,mm = 400\,mm^2$

$E = 200\,GPa$

$= 200 \times 10^9 \times 10^{-6}$

$= 200 \times 10^3\,N/mm^2$

$= 2 \times 10^5\,N/mm^2$

$F = 40\,KN$

$= 40 \times 10^3$

$= 4 \times 10^4\,N$

$$\Delta l = \frac{F \times l}{A \times E}$$

$$= \frac{4 \times 10^4 \times 1000}{400 \times 2 \times 10^5}$$

$$= \frac{4 \times 10^4 \times 10^3}{400 \times 2 \times 10^5}$$

$$= \frac{\cancel{4} \times \cancel{10^7}}{8 \times \cancel{10^7}} = \frac{1}{2} = 0.5\,mm$$

$$\boxed{\Delta l = 0.5\,mm}$$

Q. A hollow cylinder 2 m long is having outside dia of 50 mm and inside dia of 30 mm. The cylinder is subjected to a force of 25 KN. If the E = 100 GPa then calculate

(i) stress of the body

(ii) Elongation / deformation. (Δl)

solid ⇒ rod / beam / wire / cylinder $A = \frac{\pi}{4}(d)^2$

Hollow $A = \frac{\pi}{4}(D^2 - d^2)$

Given data $l = 2\,m = 2000\,mm$

$A = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}\left[(50)^2 - (30)^2\right] = \frac{\pi}{4} \times 1600$

$= \cancel{\frac{\pi}{4} \times 400 = 3.14 \times 100}$

$= 1256\,mm = 1256\,mm^2$ ~~= 314 mm²~~

$E = 100\,GPa$

$= 100 \times 10^9 \times 10^{-6}$

$= 100 \times 10^3 = 10^5\,N/mm^2$

$F = 25\,KN$

$= 25 \times 10^3\,N$

$$\text{stress} = \frac{\text{Force}}{\text{Area}} = \frac{25 \times 10^3}{1256} = 19.90\,N/mm^2$$

$$\boxed{19.90\,N/m^2}$$

$$\Delta l = \frac{F \times L}{A \times E}$$

$$= \frac{25 \times 10^3 \times 2000}{1256 \times 10^5}$$

$$= \frac{25 \times 2 \times 10^5}{1256 \times 10^5} = \frac{50}{314} = 0.1\,mm$$

$$\Delta L = \frac{F \times L}{A \times E}$$

$$= \frac{25 \times 10^3 \times 2000}{1256 \times 10^5}$$

$$= 0.398 \text{ mm.}$$

$$\boxed{\Delta L = 0.398 \text{ mm}}$$

Q. A load of 5 KN is subjected on a steel wire having the stress of 100 MPa. Calculate the diameter of the steel wire.

Data given $F = 5 \text{ KN}$

$= 5 \times 10^3 \text{ N}$

$\sigma = 100 \text{ MPa}$

$= 100 \times 10^6 \times 10^{-6} \text{ N/mm}^2$

$= 100 \text{ N/mm}^2$

$$\sigma = \frac{F}{A} = \frac{5 \times 10^3}{A}$$

$$\Rightarrow 100 = \frac{5 \times 10^3}{A}$$

$$\Rightarrow A = \frac{5 \times 10^3}{10^2}$$

$$\Rightarrow A = 5 \times 10^1$$

$$A = \frac{\pi}{4} d^2 \Rightarrow \frac{\pi}{4} d^2 = 50$$

$$\Rightarrow d^2 = \frac{50}{\pi} \times 4$$

$$d = \sqrt{\frac{50}{\pi} \times 4}$$

$$= \sqrt{\frac{200}{3.14}}$$

$$= 7.98 \text{ mm}$$

$$\boxed{d = 7.98 \text{ mm}}$$

Q. A steel rod having 13 mm diameter is having length of 200 mm. It is subjected to a tensile force of 26.8 KN which results in elongation of 0.2 mm. Calculate the value of young's Modulus for the steel rod.

Given data $d = 13 \text{ mm}$

$L = 200 \text{ mm}$

$\Delta l = 0.2 \text{ mm}$

$F = 26.8 \text{ KN}$

$= 26.8 \times 10^3 \text{ N}$ $\quad E = ?$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (13)^2$$

$$= \frac{\pi}{4} \times 169$$

$$= 132.73 \text{ mm}^2$$

$$\Delta L = \frac{F \times L}{A \times E}$$

$$\Rightarrow 0.2 = \frac{26.8 \times 10^3 \times 200}{132.73 \times A}$$

$$\Rightarrow E = \frac{26.8 \times 10^3 \times 200}{132.73 \times 0.2}$$

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$\boxed{E = 201913.65\ N/mm^2}$

$E = 201.913\ KN/mm^2$

$= 201.913 \times 10^3\ N/m^2$

$= 201.91\ KN/mm^2$

Q A hollow steel tube 3.5m long is having external dia of 120mm. It is subjected to a force of 400 KN resulting in elongation of 2mm. If E = 200 GPa calculate the internal dia of tube.

Given data $l = 3.5\ m = 3500\ mm$

$D = 120\ mm$

$F = 400\ KN$ $\quad \Delta L = 2mm$

$= 400 \times 10^3\ N$

$E = 200\ GPa$

$= 200 \times 10^9 \times 10^{-6}$

$= 200 \times 10^3$

$= 2 \times 10^5\ N/mm^2$

$$\Delta l = \frac{F \times l}{A \times E}$$

$$2 = \frac{4 \times 10^5 \times 3500}{A \times 2 \times 10^5}$$

$$\Rightarrow A = \frac{\cancel{4 \times 10^5} \times 3500}{\cancel{2 \times 2 \times 10^5}} = 3500\ mm^2$$

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$\Rightarrow \frac{\pi}{4}((120)^2 - d^2) = 3500$$

$$\Rightarrow 120^2 - d^2 = \frac{3500 \times 4}{\pi}$$

$$\Rightarrow 14400 - d^2 = 4456.33$$

$$\Rightarrow d^2 = 14400 - 4456.33$$

$$\Rightarrow d = \sqrt{9943.67}$$

$$= 99.71\ mm$$

$$\boxed{d = 99.71\ mm}$$

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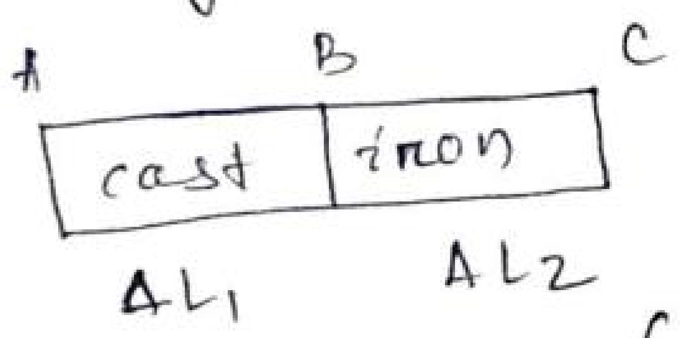
PRINCIPLE OF SUPER POSITION

Principle of super position states that the algebric sum of the deformation in individual section of the body is equal to the total deformation on elongation produce in the body.



$\Delta L = \Delta L_1 + \Delta L_2$

$\Delta L = \frac{F_1 L_1}{AE} + \frac{F_2 L_2}{AE}$

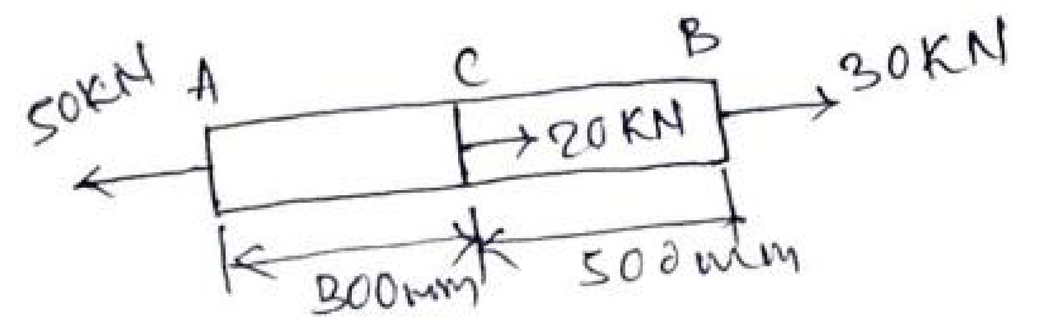
Since E = same for both parts
A = same for both parts

$\boxed{\Delta L = \frac{1}{AE}\left[F_1 L_1 + F_2 L_2\right]}$ *

⇒ similarly if the body is devided into 3 parts then

$\boxed{\Delta L = \frac{1}{AE}\left[F_1 L_1 + F_2 L_2 + F_3 L_3\right]}$ *

Q



Given $E = 200\ GPa$

$= 200 \times 10^9 \times 10^{-6}\ N/mm^2$

$= 200 \times 10^3 \Rightarrow 2 \times 10^5$

$A = 200\ mm^2$

$F_1 = 30\ KN$ $F_2 = 20\ KN$

$= 30 \times 10^3\ N$ $= 20 \times 10^3\ N$

$= 3 \times 10^4$ $= 2 \times 10^4$

$L_1 = 800mm$ $L_2 = 300mm$

$\Delta L = \frac{1}{AE}\left[(F_1 L_1 + F_2 L_2)\right]$

$= \frac{1}{200 \times 2 \times 10^5}\left[(3 \times 10^4 \times 800) + (2 \times 10^4 \times 300)\right]$

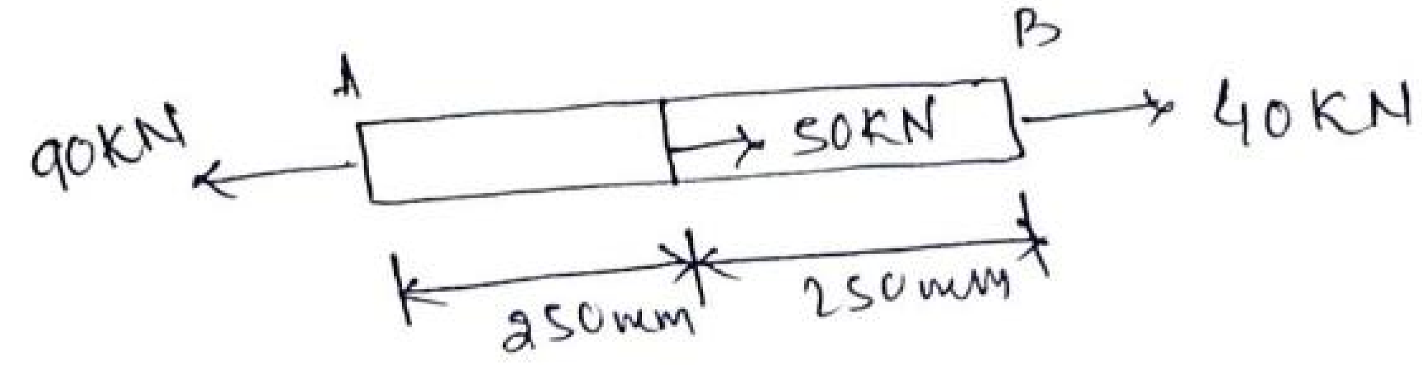
$= \frac{1}{4 \times 10^7}\left[(24 \times 10^6 + 6 \times 10^6)\right]$

$= \frac{3}{4}$

$= 0.75\ mm$

$\boxed{\Delta L = 0.75mm}$

Q

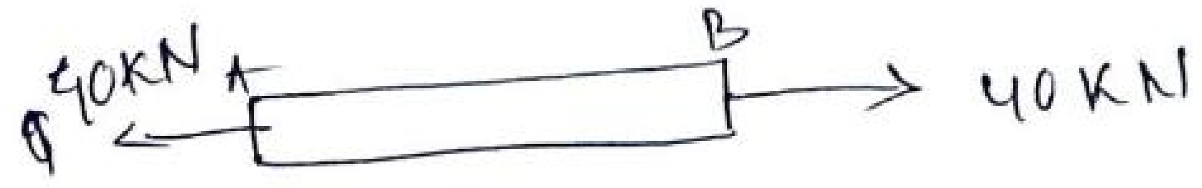


Given $E = 200\ GPa$

$= 200 \times 10^9 \times 10^{-6}\ N/mm^2$

$= 2 \times 10^5$

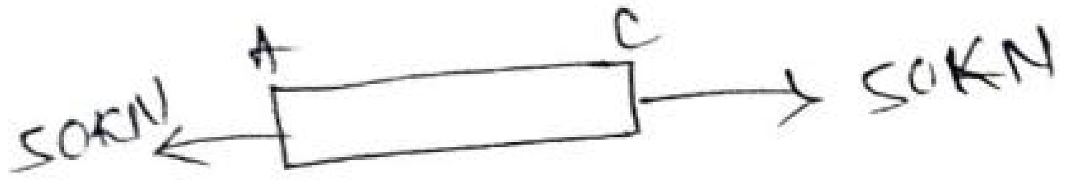
$A = 300\ mm^2$



$F_1 = 40\ KN$ $L_1 = 250 + 250$

$= 40 \times 10^3\ N$ $= 500\ mm$

$= 4 \times 10^4\ N$



$F_2 = 50\ KN$ $L_2 = 250\ mm$

$= 5 \times 10^4\ N$

$$\Delta L = \frac{1}{AE}\left[(F_1L_1 + F_2L_2)\right]$$

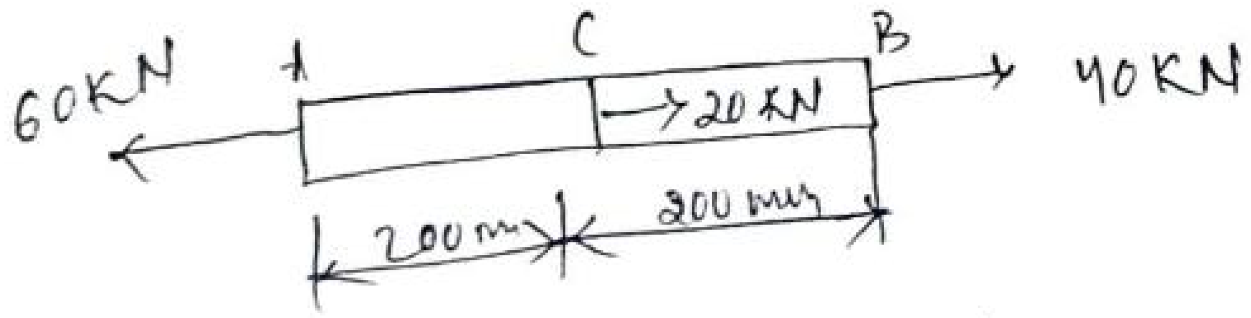
$$= \frac{1}{300 \times 2 \times 10^5}\left[4 \times 10^4 \times 500 + 5 \times 10^4 \times 250\right]$$

$$= \frac{1}{\cancel{3000} \; 6 \times 10^7}\left[20 \times 10^6 + 5 \times 10^4 \times 250\right]$$

$= 0.54\ mm$

$\boxed{\Delta L = 0.54\ mm}$

Q



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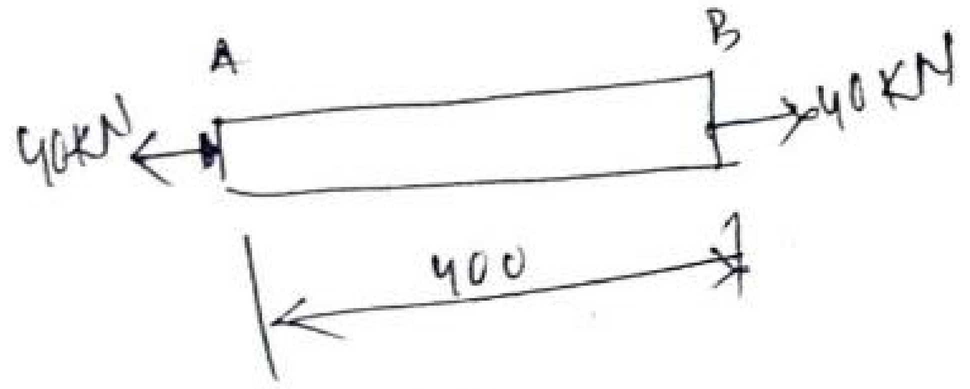

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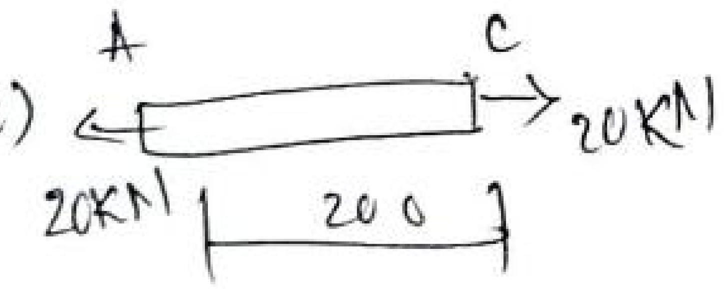


$F_1 = 40\ KN$

$= 4 \times 10^4\ N$

$L_1 = 400\ mm$

$F_2 = 20\ KN$

$= 2 \times 10^4\ N$

$L_2 = 200\ mm$

Given $E = 200\ GPa$

$= 200 \times 10^9 \times 10^{-6}\ N/mm^2$

$= 200 \times 10^3 \Rightarrow 2 \times 10^5\ N/mm^2$

$d = 20\ mm$

find $\Delta L = ?$

$$A = \frac{\pi}{4}(d)^2 \Rightarrow \frac{\pi}{4} \times \cancel{400}\,100 = 314.15\ mm^2$$

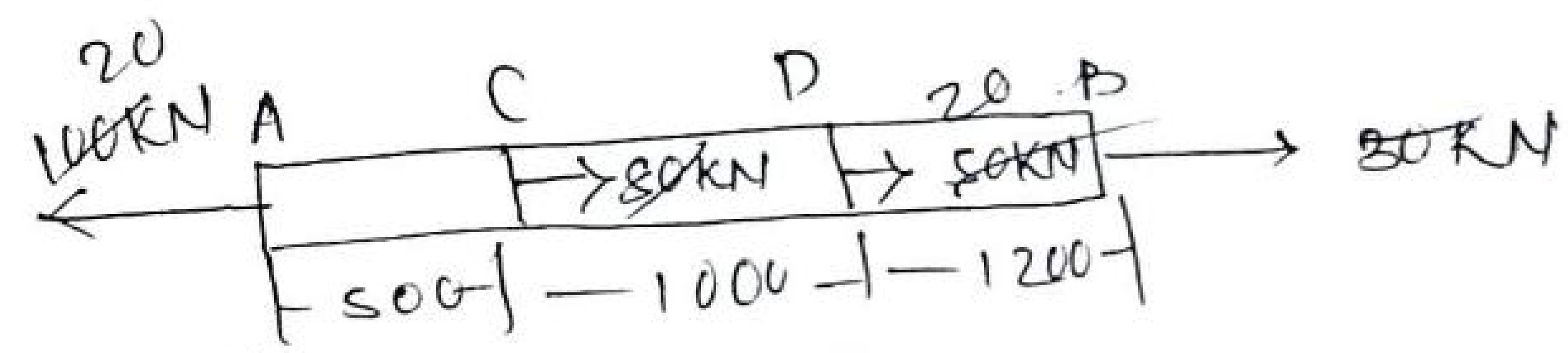
$$\Delta L = \frac{1}{AE}\left[F_1L_1 + F_2L_2\right]$$

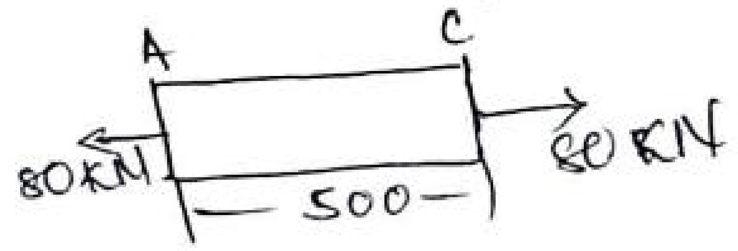
$$= \frac{1}{314.15 \times 2 \times 10^5}\left[4 \times 10^4 \times 400 + 2 \times 10^4 \times 200\right]$$

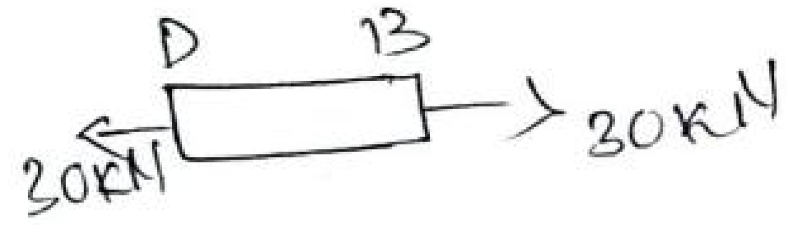
$= 0.31\ mm$

$\boxed{\Delta L = 0.31\ mm}$

Q







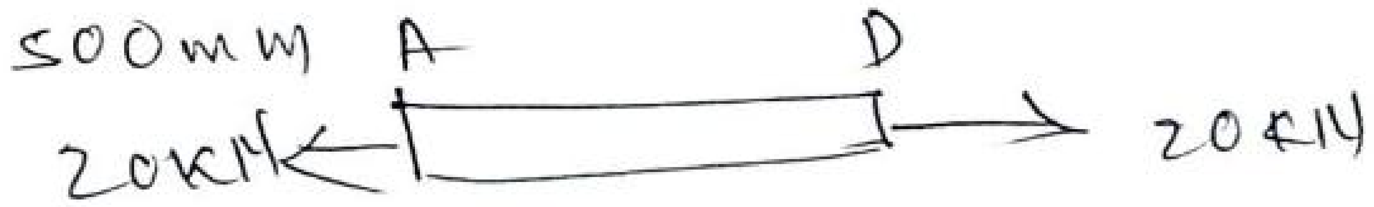
$F_1 = 80\ KN$

$= 8 \times 10^4$

$L_1 = 500\ mm$

$F_2 = 30\ KN$ $L_2 = 1200\ mm$

$= 3 \times 10^4\ N$



$F_3 = 20\ KN \Rightarrow 2 \times 10^4$

$L_3 = \cancel{1500}\ 1500\ mm$

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Given $E = 800\ GPa$

$= 800 \times 10^3\ N/mm^2$

$= 8 \times 10^5\ N/mm^2$

$A = 500\ mm^2$

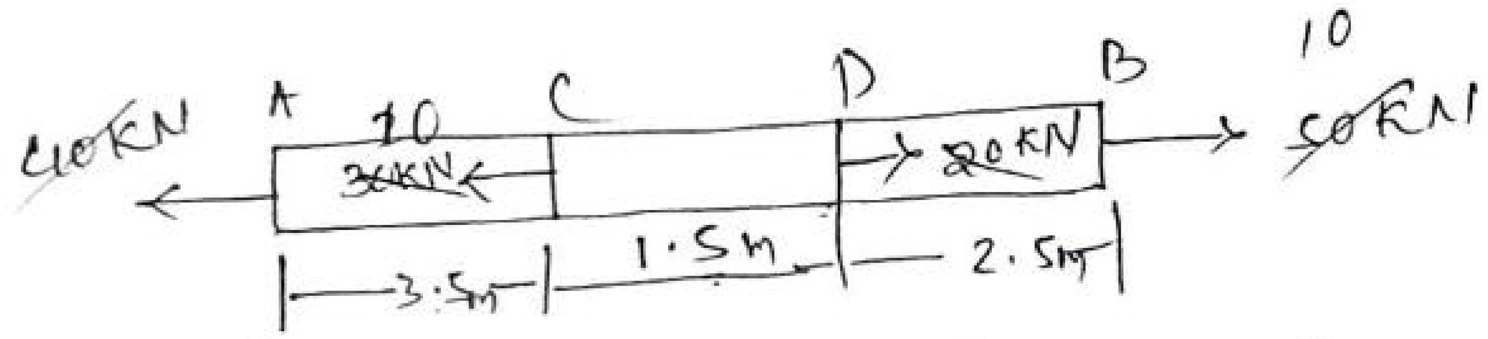
$$\Delta_L = \frac{1}{AE}\left[F_1L_1 + F_2L_2 + F_3L_3\right]$$

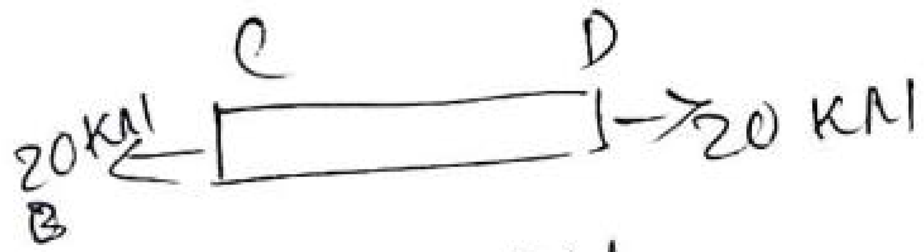
$$= \frac{1}{500 \times 8 \times 10^5}\left[8\times10^4\times500 + 3\times10^4\times1200 + 2\times10^4\times1500\right]$$

$= 2.65\ mm$

$\boxed{\Delta_L = 2.65\ mm}$

Q





$F_1 = 40KN$

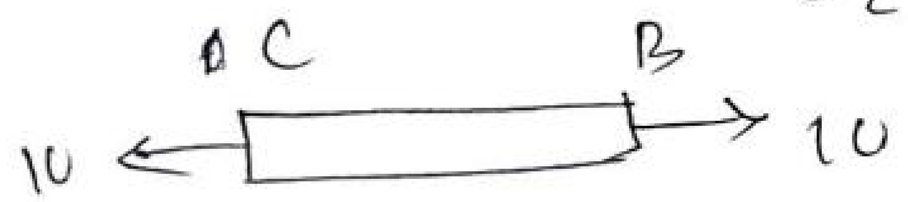
$= 4\times10^4\ N$

$L_1 = 7500\ mm$

$F_2 = 20KN$

$= 2\times10^4\ N$

$L_2 = 1500\ mm$



$F_3 = 10\ KN = 1\times10^4$

$L_3 = 4000\ mm$

Given $E = 100\ GPa \Rightarrow 100\times10^3 \Rightarrow 1\times10^5\ N/mm^2$

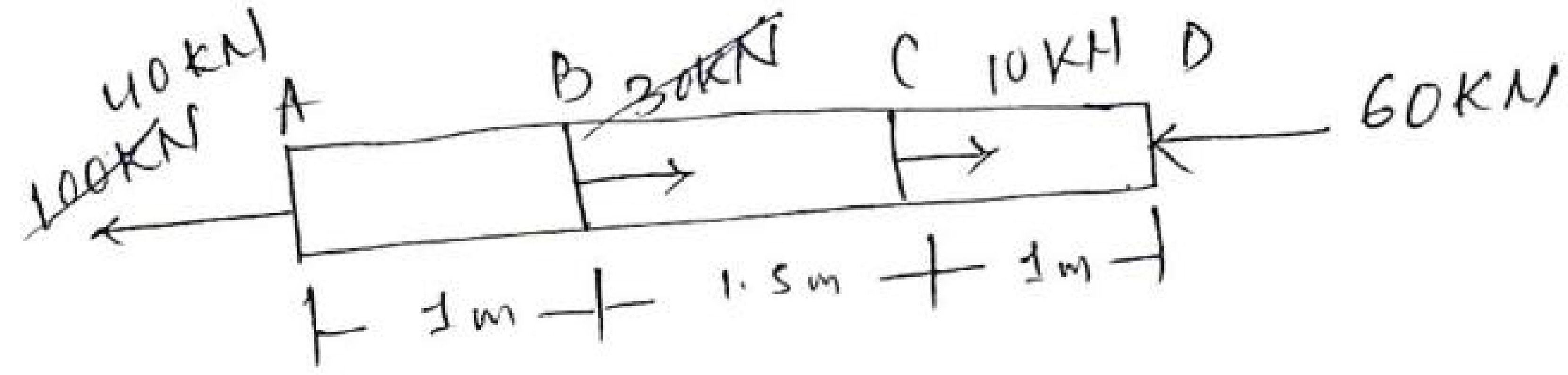
$A = 800\ mm^2$

$$\Delta_L = \frac{1}{10^5 \times 800}\left[4\times10^4\times7500 + 2\times10^4\times1500 + 10^4\times4000\right]$$

$\Delta_L = 4.625\ mm$

$\boxed{\Delta_L = 4.625\ mm}$

Q



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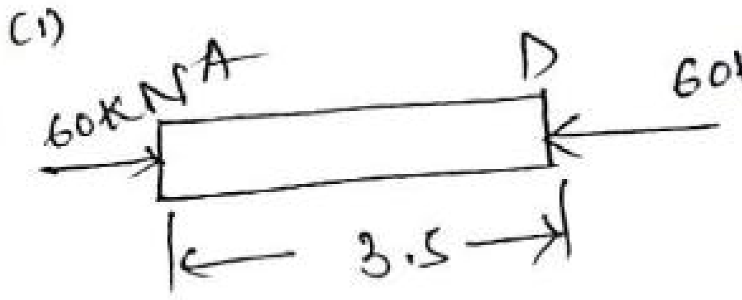

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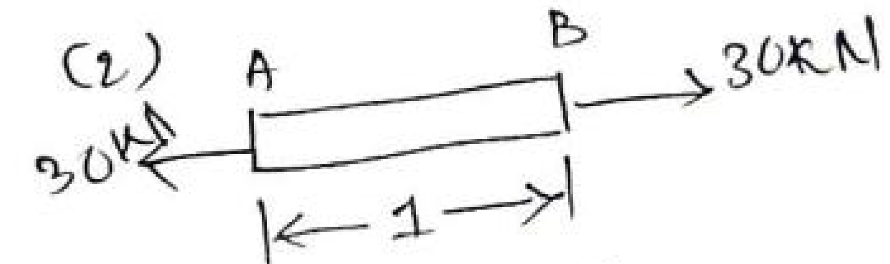


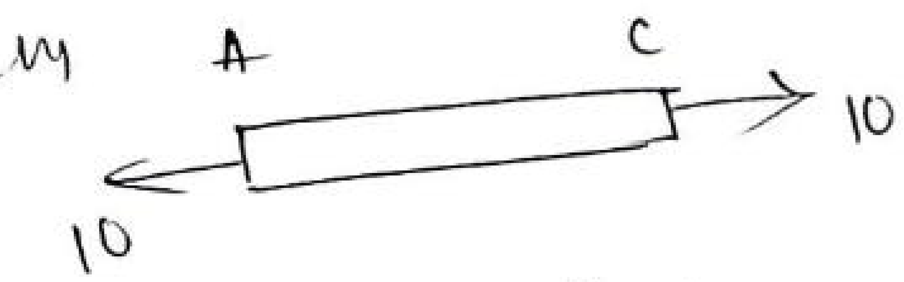
$F_1 = -60KN$

$= -6\times10^4\ N$

$L_1 = 3500\ mm$

$F_2 = 3\times10^4\ N$

$L_2 = 1000\ mm$



$F_2 = 1\times10^4\ N$

$L_3 = 2500\ mm.$

Given $A = 500\ mm^2$

$E = 200\ GPa = 200\times10^3 \Rightarrow 2\times10^5\ N/mm^2$

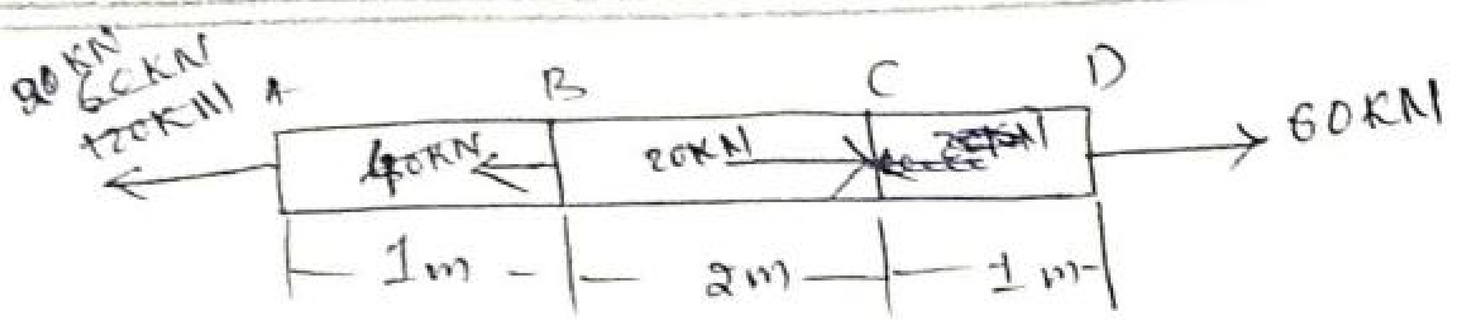
$\Delta_L = ?$

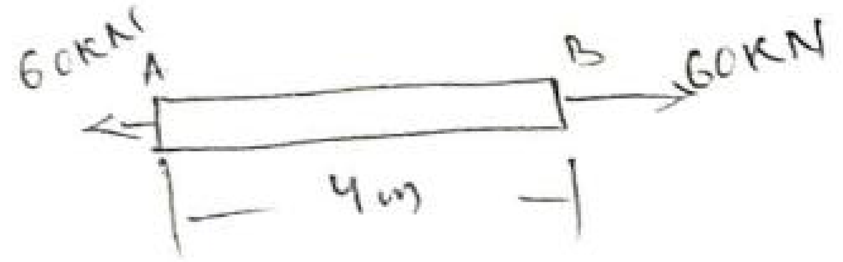
$$\Delta_L = \frac{1}{500\times2\times10^5}\left[-6\times10^4\times3500 + 3\times10^4\times1000 + 10^4\times2500\right]$$

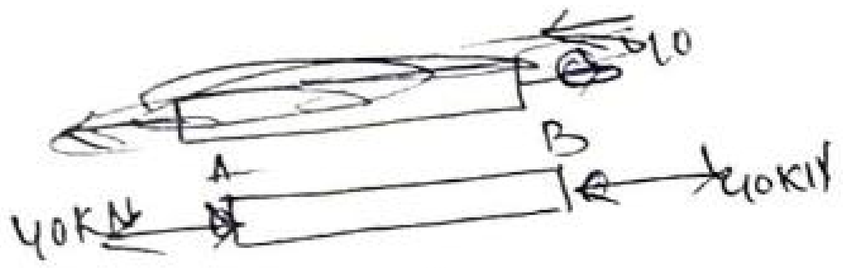
$= -1.55\ mm$

$\boxed{\Delta_L = -1.55\ mm}$

Q

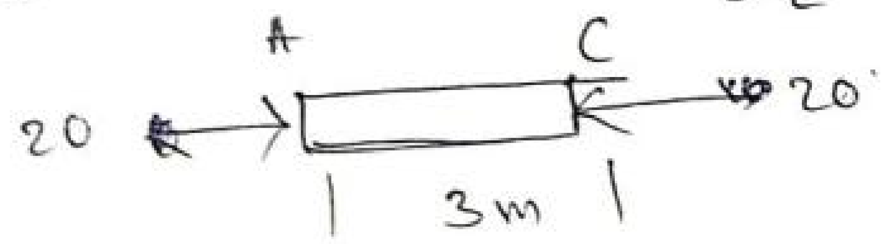






$F_1 = 6 \times 10^4$ N $\qquad F_2 = 4 \times 10^4$ N

$L_1 = 4000$ mm $\qquad L_2 = 1000$ mm



$F_3 = -2 \times 10^4$ N

$L_3 = 3000$ mm

Given $E = 100$ GPa $\Rightarrow 1 \times 10^5$ N/m2

$A = 400$ mm2

$$\Delta l = \frac{1}{400 \times 10^5}\left[6\times10^4\times4000 + 4\times10^4\times1000 - 2\times10^4\times3000\right]$$

$= 5.5$ mm

$\boxed{\Delta l = 5.5 \text{ mm}}$

LOAD

Load can be defined as an external agent that is capable of applying force on the surface of the any body to which they are applied.

TYPES OF LOADS:-

There are four types of load. They are

1) Dead load
2) Variable load
3) Suddenly applied load
4) Impact load.

1) **DEAD LOAD:-** The load which can't change it's position with respect to time and surrounding is known as dead load

2) **VARIABLE LOAD:-** The load which can change it's position with respect to time and surrounding is known as Variable load.

3) **SUDDENLY APPLIED LOAD:-** The loads which are applied in a very short interval of time or which are applied instantaneously are called suddenly applied load.

instantaneously → short interval of time.

4) **IMPACT LOAD**:— The load which is having an initial velocity is called an impact load.
ex:- catching of a ball having initial velocity.

COMPOSITE SECTION (More than one)

Composite section can be defined as the section on the body which are having more than one material.

STRESSES IN COMPOSITE SECTION:—

1) $E = \frac{\sigma}{\varepsilon}$

Material 1

$E_1 = \frac{\sigma_1}{\varepsilon_1}$

$\Rightarrow \varepsilon_1 = \frac{\sigma_1}{E_1}$

Material-2

$E_2 = \frac{\sigma_2}{\varepsilon_2}$

$\Rightarrow \varepsilon_2 = \frac{\sigma_2}{E_2}$

Material 1
material 2

strain = same

$\therefore \varepsilon_1 = \varepsilon_2$

$\Rightarrow \boxed{\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}}$

(Strain are same for both the material)

2) Stress = $\frac{\text{Force}}{\text{Area}}$

$\Rightarrow \sigma = \frac{F}{A}$

$\Rightarrow F = \sigma A$

For material 1 $\Rightarrow F_1 = \sigma_1 \times A_1$

material 2 $\Rightarrow F_2 = \sigma_2 \times A_2$

Total force on composite section (F)

$F = F_1 + F_2$

$\boxed{F = (\sigma_1 A_1) + (\sigma_2 A_2)}$

Q A concrete circular section of 50000 mm² contain 6 steel bars having cross-sectional area of 500 mm². Calculate the load on the column if the ~~Strength~~ stress of the concrete is given 3.5 MPa. Take the ratio of young's modulus of steel is 18

Ans Data given:—

Total area = 50000 mm²

Area of steel = 500 mm²

stress of concrete σ_c = 3.5 MPa
= 3.5 N/mm²

$\frac{E_s}{E_c} = 18$ F = ?

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Area of concrete = Total area − area of steel

$= 50000 - 500$

$= 49500\ mm^2$

$\therefore \dfrac{\sigma_1}{E_1} = \dfrac{\sigma_2}{E_2}$ (strain is equal for both the body so $\frac{\sigma}{E}$)

$\Rightarrow \dfrac{\sigma_s}{E_s} = \dfrac{\sigma_c}{E_c}$

$\Rightarrow \sigma_s = \dfrac{\sigma_c \times E_s}{E_c}$

$\Rightarrow \sigma_s = 3.5 \times 18$

$\Rightarrow \sigma_s = 63\ N/mm^2$

$F = F_1 + F_2$

$= \sigma_s A_s + \sigma_c A_c$

$= 63 \times 500 + 49500 \times 3.5$

$= 204750\ N$

$= 204.750\ KN$

Q. A concrete rectangular column 500 x 500 mm in cross-sectional contains 4 steel bar. The dia of each steel bar is 25 mm. Calculate the stress in the concrete and steel bars if the total force on the section is given 1000 KN Taken $E_s = 210$ GPa and $E_c = 14$ GPa.

Ans

Data given:-

Total area = $500 \times 500 = 250000\ mm^2$

Dia of steel (d) = 25 mm

Area of single steel $= \frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times (25)^2$

$= 490.87\ mm^2$

Area of all steel rods $= 4 \times 490.87$

$= 1963.48\ mm^2$

$E_s = 210 GPa = 210 \times 10^9 \times 10^{-6}$

$= 210 \times 10^3\ N/mm^2$

$E_c = 14 GPa = 14 \times 10^9 \times 10^{-6}$

$= 14 \times 10^3\ N/mm^2$

Total Force (F) = 1000 KN

$= 1000 \times 10^3$ N

Area of concrete = Total area − Area of steel

$= 250000 - 1963.48$

$= 248036.52\ mm^2$

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$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\Rightarrow \sigma_s = \frac{\sigma_c \times E_s}{E_c}$$

$$\Rightarrow \sigma_s = \sigma_c \times \frac{\cancel{210 \times 10^3}}{\cancel{14 \times 10^3}}$$

$$\Rightarrow \sigma_s = 15\sigma_c$$

$$F = F_1 + F_2$$

$$\Rightarrow F = \sigma_s A_s + \sigma_c A_c$$

$$\Rightarrow 1000 \times 10^3 = (15\sigma_c \times 1963.48) + (\sigma_c \times 248036.52)$$

$$= 29452.35\sigma_c + 248036.52\sigma_c$$

$$= 277488.86\sigma_c$$

$$\Rightarrow \sigma_c = \frac{1000 \times 10^3}{277488.86}$$

$$= 3.60 \text{ N/mm}^2$$

$$\Rightarrow \sigma_s = 15\sigma_c$$

$$= 15 \times 3.6$$

$$= 54 \text{ N/mm}^2$$

Q. A square column having 400mm x 400mm in cross-section contains 6 steel bars having 15 mm diameter each. Calculate the ~~stress~~ stress on steel and concrete if $E_s = 200 GPa$ and $E_c = 20 GPa$ and total force = 150 KN

Ans Given data:-

Total Area: $400 \times 400 = 160000 \text{ mm}^2$

Dia of steel bar (d) = 15mm

Area of steel bar $= \frac{\pi}{4}(d)^2$

$= \frac{\pi}{4} \times (15)^2 = 176.71 \text{ mm}^2$

Total area of steel bar $= 4 \times 176.71$

$= 706.84 \text{ mm}^2$

Area of concrete = Total area − Area of steel

$= 160000 - 706.84$

$= 159293.16 \text{ mm}^2$

$E_s = 200 GPa = 200 \times 10^9 \times 10^{-6}$

$= 200 \times 10^3 = 2 \times 10^5 \text{ N/mm}^2$

$E_c = 20 GPa = 20 \times 10^9 \times 10^{-6}$

$= 20 \times 10^3$

$= 2 \times 10^4 \text{ N/mm}^2$

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$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\Rightarrow \sigma_s = \frac{\sigma_c \times E_s}{E_c}$$

$$\Rightarrow \sigma_s = \sigma_c \times \frac{2 \times 10^5}{2 \times 10^4}$$

$$\Rightarrow \sigma_s = 10\sigma_c$$

$F = F_1 + F_2$ (Total force = 150KN = 150×10^3 N)

$\Rightarrow 150 \times 10^3 = \sigma_s A_s + \sigma_c A_c$

$\Rightarrow 150 \times 10^3 = (10\sigma_c \times 706.84) + (\sigma_c \times 159293.16)$

$\Rightarrow 150 \times 10^3 = 7068.4\sigma_c + 159293.16\sigma_c$

$$\Rightarrow \sigma_c = \frac{150 \times 10^3}{166361.56}$$

$\Rightarrow \sigma_c = 0.9$ N/mm²

$\sigma_s = 10\sigma_c$
$= 10 \times 0.9 = 9$ N/mm²

Q. A concrete column 400 mm diameter has 4 still bar having 200 mm diameter each. Find the maximum load if $\sigma_s = 120$ MPa and $\sigma_c = 5$ MPa. Take the ratio young's modulos of steel to young's modulos of concrete is 10.

Ans

Given data :- $D = 400$ mm

$$\text{Total Area} = \frac{\pi}{4} d^2$$
$$= \frac{\pi}{4} \times (400)^2$$
$= \cancel{12\,4800}$ 125663.7 mm²

$$\text{Area of steel} = \frac{\pi}{4} (d)^2 \times 4$$
$$= \frac{\pi}{4} \times (20)^2 \times 4$$
$= 1256.63$

Area of concrete = Total Area − Area of steel
$= 125663.7 - 1256.63$
$= 124407.07$ mm.

$F = F_1 + F_2$
$= \sigma_s A_s + \sigma_c A_c$
$= 120 \times 1256.63 + 5 \times 124407.07$
$= 772830.95$ N

Q. The diameter of each steel ~~rod~~ is 25 mm Calculate the stress in the concrete and steel bar if the total force on the section is given 1000KN. Take $E_s = 200\,GPa$ and $E_c = 14\,GPa$. Area of section $500 \times 500\,mm$

Ans Given data:- T.A = $500 \times 500\,mm$
$= 250000\,mm^2$

$d = 25\,mm$

Area of steel $= \frac{\pi}{4} \times d^2$
$= \frac{\pi}{4} \times (25)^2$
$= 490.87\,mm^2$

Area of concrete = Total Area - Area of steel
$= 250000 - 490.87$
$= 249509.13\,mm^2$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$\Rightarrow \sigma_s = \frac{\sigma_c \times E_s}{E_c}$

$\Rightarrow \sigma_s = \sigma_c \times \frac{2 \times 10^5}{14 \times 10^3}$

$\Rightarrow \frac{\sigma_s}{\sigma_c} = \frac{100}{7} = 14.28\,\sigma_c$

$E_c = 14\,GPa$
$= 14 \times 10^3\,N/mm^2$

$E_s = 200\,GPa$
$= 200 \times 10^3\,N/mm^2$
$= 2 \times 10^5\,N/mm^2$

$F = 1000\,KN$
$= 10^3 \times 10^3\,N$
$= 10^6\,N$

$F = F_s + F_c$

$\Rightarrow 10^6 = \sigma_s A_s + \sigma_c A_c$

$\Rightarrow 10^6 = (15\sigma_c \times 1963.78) + \sigma_c \times 248036.52)$

$\Rightarrow 10^6 = 29452.2\sigma_c + 248036.52\sigma_c$

$\Rightarrow 10^6 = 277488.72\,\sigma_c$

$\Rightarrow \sigma_c = \frac{1000000}{277488.72} = 3.60\,N/mm^2$

$\sigma_s = 15\sigma_c$
$= 15 \times 3.6 = 54\,N/mm^2$

TEMPERATURE STRESS / THERMAL STRESS:-

⇒ When the body is subjected to tensile stress the length of the body is increase. Whenever the body is subjected compressive stress, than the ~~body~~ length of the body decreases.

⇒ When a body restricted to any expansion or contraction than a stress is induced in the body and this stress is known as thermal stress or temperature stress.

THERMAL STRESS IN SIMPLE BARS:-

Yeild - deform - change in shape and size.

Thermal stress in simple bars. Thermal stress in calculate from this method.

Case-1 No Yeild ~~strong~~ conditions

$$\boxed{\sigma_t = \alpha t E}$$

where

σ_t = Temperature stress.

t = temperature.

E = Young's Modulus

α = Co-efficient of linear expansion.

Case-2 Yield condition

$$\sigma_t = \left[\alpha t - \frac{\Delta}{l}\right] E$$

where σ_t = temperature stress

α = co-efficient of linear expansion

Δ = yield.

l = length of body.

E = Young's Modulus.

Q. A circular bar is heated to a temperature of 20°K. Find out the thermal stress in duested in the body in if young's modulous in bar 80 GPa and co-efficient of linear expansion is 24×10^{-6} K

Ans Given $t = 20K$

$\alpha = 24\times10^{-6}$ K

$E = 80$ GPa

$= 80\times10^{9}\times10^{-6}$ N/mm²

$= 80\times10^{3}$ N/mm²

$\sigma_t = \alpha t E$

$= 24\times10^{-6}\times20\times80\times10^{3}$

$= 38.4$ N/mm²

Q. A brass rod is subjected to a temperature stress of 76.5 MPa. Calculate the temperature through which the rod heated if E = 90GPa and $\alpha = 17\times10^{-6}$/K

Ans Data given $\sigma_t = 76.5$ GPa

$= 76.5$ N/mm²

$E = 90$ GPa

$= 90\times10^{3}$ N/mm²

$\alpha = 17\times10^{-6}$ t

$$\sigma_t = \alpha t E$$

$$\Rightarrow 76.5 = 17\times10^{-6} \times 90\times10^{3}\times t$$

$$\Rightarrow t = \frac{76.5}{17\times10^{-6}\times 90\times10^{3}}$$

$$= 50\,K$$

Q. A rod of 6m a fix at the both end. It is heated to a temperature of 40°c. Calculate the thermal stress devolved in the bar if the yield is given 1 m. Take E = 200 GPa and $\alpha = 12\times10^{-6}/°c$

Data given :- Length of rod = 6m = 6×1000 = 6000 mm.

$t = 40°c$

(Δ) yield = 1 mm

$E = 200\,GPa = 200\times10^{3}\ N/mm^2$

$\alpha = 12\times10^{-6}/°c$

$$\sigma_t = \left[\alpha t - \frac{\Delta}{l}\right]E$$

$$= \left[12\times10^{-6}\times 40 - \frac{1}{6000}\right]\times 200\times10^{3}$$

$$= 62.66\ N/mm^2$$

Q. A steel bar fixed at both ends. It heated through 15K. Calculate the stress developed in the bar. If modulus of elasticity and co-efficient of linear expansion ~~are~~ for the bar material is 200 GPa & $12\times10^{-6}/K$ respectively.

Ans. Data given $t = 15K$

$\alpha = 12\times10^{-6}/K$

$E = 200\,GPa = 200\times10^{3}\ N/mm^2$

$$\sigma_t = \alpha t E$$

$$= 12\times10^{-6}\times 15\times 200\times10^{3}$$

$$= 36\,K$$

Q. A steel bar is heated to a temperature of 50K. Calculate the thermal stress induced in the steel bar. Take $\alpha = 12\times10^{-6}$ K, length $l = 5m$ & E = 200 GPa

(i) No yield case

(ii) Yield case / take $\Delta = 2mm$

Data given :- $t = 50K$

$l = 5m = 5\times1000 = 5000\,mm$

$\alpha = 12\times10^{-6}\,K$

$\Delta = 2\,mm.$

$E = 200\times10^{3}\ N/mm^2$

i) No yield case.

$$\sigma_t = \alpha t E$$
$$= 12 \times 10^{-6} \times 50 \times 200 \times 10^3$$
$$= 120 K$$

ii) Yield case

$$\sigma_t = \left[\alpha t - \frac{A}{\ell}\right] E$$
$$= \left[12 \times 10^{-6} \times 50 - \left(\frac{1}{5000}\right)\right] 200 \times 10^3$$
$$= 40 \ N/mm^2$$

TEMPERATURE STRESS IN COMPOSITE BAR

- When a composite bar section are subjected to temperature than components of bar will expand to same extent. This change in length with occur in both the elements of the bar but changes will not be same.
- Steps to calculate thermal stress in composite bars.

Step-1 → calculate of areas of both the components.

Area = $W \times t$ (Rectangular)

Area = $\frac{\pi}{4}(d)^2$ (Circular)

Step-2 → Calculation of strain & relation between stress.

Young's Modulus = $\frac{\text{stress}}{\text{strain}}$.

$$\Rightarrow E = \frac{\sigma}{\varepsilon}$$
$$\varepsilon = \frac{\sigma}{E}$$

$$\boxed{\therefore \varepsilon_1 = \frac{\sigma_1}{E_1}} \qquad \boxed{\varepsilon_2 = \frac{\sigma_2}{E_2}}$$

$$\sigma = \frac{F}{A} \Rightarrow F = \sigma A \Rightarrow F_1 = \sigma_1 A_1 \quad \text{---(i)}$$
$$F_2 = \sigma_2 A_2 \quad \text{---(ii)}$$
$$\therefore F_1 = F_2$$
$$\Rightarrow \sigma_1 A_1 = \sigma_2 A_2$$
$$\Rightarrow \boxed{\sigma_1 = \frac{\sigma_2 A_2}{A_1}}$$

Step-3 → Calculation of stress.

$$\boxed{\varepsilon_1 + \varepsilon_2 = t(\alpha_1 - \alpha_2)}$$

α = co-efficient of linear expansion

1°C ↓↑ = metal expand / contract

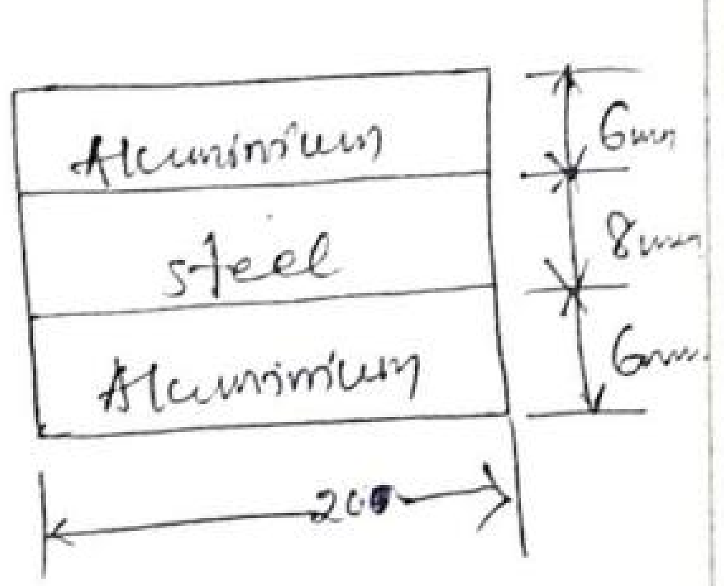
Q. Calculate the stress on the steel aluminium bar present inside a composite bar as shown in the figure. The composite bar is heated to temperature of 50°C Take

$E_A = 80$ GPa

$E_S = 200$ GPa

$\alpha_A = 24 \times 10^{-6} /°C$

$\alpha_S = 12 \times 10^{-6} /°C$



Ans. Given data :-

$E_A = 80$ GPa

$= 80 \times 10^3$ N/mm2

$E_S = 200$ GPa

$= 2 \times 10^5$ N/mm2 / 200×10^3 N/mm2

Area of steel = $W \times t = 20 \times 8 = 160$ mm2

Area of Aluminium = $w \times t \times 2$

$= 20 \times 6 \times 2$

$= 240$ mm2

$\epsilon = \frac{\sigma}{E} = \epsilon_S = \frac{\sigma_S}{E_S} = \frac{\sigma_S}{200 \times 10^3}$

$\epsilon_{Al} = \frac{\sigma_{Al}}{E_{Al}} = \frac{\sigma_{Al}}{80 \times 10^3}$

$F_1 = F_2$

$\Rightarrow \sigma_1 A_1 = \sigma_2 A_2$

$\sigma_{Al} A_{Al} = \sigma_S A_S$

$\Rightarrow \sigma_{Al} = \frac{\sigma_S A_S}{A_{Al}} = \frac{\sigma_S \times \cancel{160}^{\cancel{4}\,2}}{\cancel{240}_{\cancel{6}\,3}}$

$\Rightarrow \sigma_{Al} = \sigma_S \times 0.66$

$\therefore \epsilon_1 + \epsilon_2 = t(\alpha_1 - \alpha_2)$

$\Rightarrow \frac{\sigma_S}{200 \times 10^3} + \frac{\sigma_{Al}}{80 \times 10^3} = 50(24 \times 10^{-6} - 12 \times 10^{-6})$

$\Rightarrow \frac{\sigma_S}{200 \times 10^3} + \frac{\sigma_{Al}}{80 \times 10^3} = 50 \times 10^{-6} (12)$

$\Rightarrow \sigma_S \left(\frac{1}{200 \times 10^3} + \frac{0.66}{80 \times 10^3} \right) = 50 \times 12 \times 10^{-6}$

$\Rightarrow \sigma_S \times 1.325 \times 10^{-5} = 6 \times 10^{-4}$

$\Rightarrow \sigma_S = \frac{6 \times 10^{-4}}{1.325 \times 10^{-5}} = 45.28$ N/mm2

$\Rightarrow \sigma_{Al} = 0.66 \times \sigma_S$

$= 0.66 \times 45.28$

$= 29.88$ N/mm2

Short

1) Area $= w \times t$

$= \frac{\pi}{4} d^2$

2) $\epsilon_1 = ?$ & $\epsilon_2 = ?$

σ_1 & σ_2 = relation

$\Rightarrow \epsilon_1 + \epsilon_2 = t\,(\alpha_1 - \alpha_2)$

Value put

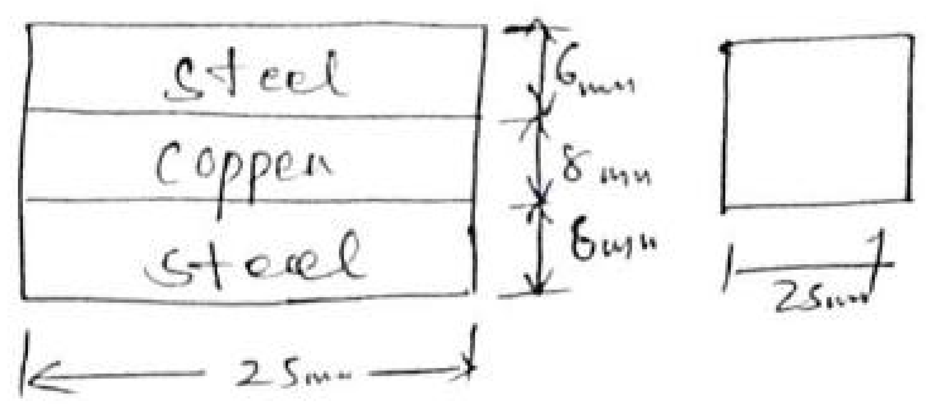
stress relation put

4) $\sigma_1 = ?$

$\sigma_2 = ?$

3. A composite bar is subjected to a temperature of 50°C. Calculate the stress in both the material for the figure given below.

Take
$E_s = 200\ GPa$
$E_c = 80\ GPa$
$\alpha_s = 12\times10^{-6}/°C$
$\alpha_c = 24\times10^{-6}/°C$



Data given :- $t = 50°C$

$E_s = 200\,GPa = 200\times10^3\ N/mm^2$

$E_c = 80\,GPa = 80\times10^3\ N/mm^2$

$\alpha_s = 12\times10^{-6}/°C$

$\alpha_c = 24\times10^{-6}/°C$

Area of steel = $w\times t\times 2 = 25\times6\times2$
$= 150\times2 = 300\ mm^2$

Area of copper = $w\times t = 25\times8$
$= 200\ mm^2$

$\epsilon = \frac{\sigma}{E} \Rightarrow \epsilon_c = \frac{\sigma_c}{E_c} = \frac{\sigma_c}{80\times10^3}$

$\epsilon_s = \frac{\sigma_s}{E_s} = \frac{\sigma_s}{200\times10^3}$

$\Rightarrow F_a \Rightarrow F_1 = F_c$
$\sigma_s A_s = \sigma_c A_c$

$\Rightarrow \sigma_s = \frac{\sigma_c A_c}{A_s} = \frac{\sigma_c\times200}{300}$

$\Rightarrow \sigma_s = 0.66\,\sigma_c$

$\therefore\ \epsilon_c + \epsilon_s = t(\alpha_c - \alpha_s)$

$\Rightarrow \frac{\sigma_c}{80\times10^3} + \frac{\sigma_s}{200\times10^3} = 50(24\times10^{-6} - 12\times10^{-6})$

$\Rightarrow \frac{\sigma_c}{80\times10^3} + \frac{0.66\,\sigma_c}{200\times10^3} = 50(24\times10^{-6} - 12\times10^{-6})$

$\Rightarrow \sigma_c\left(\frac{1}{80\times10^3} + \frac{0.66}{200\times10^3}\right) = 50\times10^{-6}\times12$

$\Rightarrow \sigma_c\times1.58\times10^{-5} = 6\times10^{-4}$

$\Rightarrow \sigma_c = \frac{6\times10^{-4}}{1.58\times10^{-5}}$

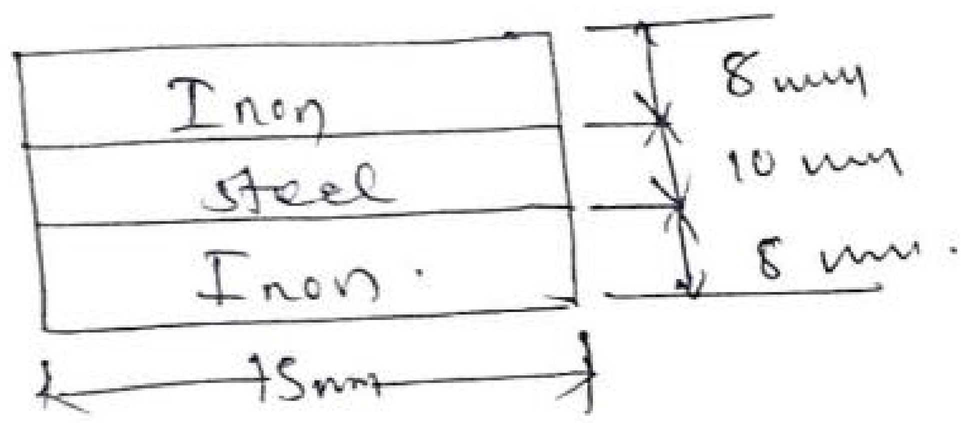
$\Rightarrow \sigma_c = 37.97\ N/mm^2$

$\Rightarrow \sigma_s = 0.66\,\sigma_c$

$\Rightarrow \sigma_s = 0.66 \times 39.97$

$\Rightarrow \sigma_s = 25.06\ N/mm^2$

Q. A composite section subjected to a temperature of 65 K calculate the stress in both the material for the figure given below



Data given $t = 65K$

$E_I = 100\ GPa$

$= 100 \times 10^3\ N/mm^2$

$E_s = 200\ GPa$

$= 200 \times 10^3\ N/mm^2$

$\alpha_1 = 36 \times 10^{-6}\ /K$

$\alpha_B = 12 \times 10^{-6}\ /K$

Area of Iron $= w \times t \times 2$

$= 15 \times 8 \times 2 = 240\ mm^2$

Area of steel $= 15 \times 10 = 150\ mm^2$

$\varepsilon = \dfrac{\sigma}{E}$

$\varepsilon_I = \dfrac{\sigma_I}{E_I} = \dfrac{\sigma_I}{100 \times 10^3}$

$\varepsilon_s = \dfrac{\sigma_s}{E_s} = \dfrac{\sigma_s}{200 \times 10^3}$

$\sigma_I A_I = \sigma_s A_s$

$\Rightarrow \sigma_I = \dfrac{\sigma_s A_s}{A_I} = \dfrac{\sigma_s \times \cancel{150}^{5}}{\cancel{240}_{8}}$

$\Rightarrow \sigma_I = 0.625\,\sigma_s\ N/mm^2$

$\therefore \varepsilon_1 + \varepsilon_2 = t(\alpha_1 - \alpha_2)$

$\varepsilon_I + \varepsilon_s = t(\alpha_I - \alpha_s)$

$\Rightarrow \dfrac{\sigma_I}{100 \times 10^3} + \dfrac{\sigma_s}{200 \times 10^3} = 65(36 \times 10^{-6} - 12 \times 10^{-6})$

$\Rightarrow \dfrac{0.625\,\sigma_s}{100 \times 10^3} + \dfrac{\sigma_s}{200 \times 10^3} = 65 \times 10^{-6} \times 24$

$\Rightarrow \sigma_s\left(\dfrac{0.625}{100 \times 10^3} + \dfrac{1}{200 \times 10^3}\right) = 65 \times 10^{-6} \times 24$

$\Rightarrow \sigma_s \times 1.125 \times 10^{-5} = 65 \times 10^{-6} \times 24$

$\Rightarrow \sigma_s = \dfrac{65 \times 10^{-6} \times 24}{1.125 \times 10^{-5}}$

$\Rightarrow \sigma_s = 138.66\ N/mm^2$

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$\sigma_I = 0.625\sigma_3$

$= 0.625 \times 138.66$

$= 86.66\ N/mm^2$

STRAIN ENERGY

→ When elastic body is loaded. It stores the energy and when the deforming force is remove and it is again regains original shape and size.

→ The energy required by the elastic body to regains its original shape and size on the energy stored in the elastic body is known as strain energy.

RESILIENCE

The strain energy store in the body is called as Resilience.

PROOF - RESILIENCE

The total strain Energy that can stored in a body is known as proof-resilience.

MODULUS OF RESILIENCE

The strain energy per unit volume is known as modulus of resilience.

$$\text{Modulus of resilience} = \frac{\text{strain Energy}}{\text{Volume.}}$$

(Case-1)

→ strain energy store in a body when is subjected gradual load.

strain energy (U)

$$U = \frac{\sigma^2}{2E} \times \text{Volume}$$

2) Volume

$= \text{Area} \times \text{Length}$

$\boxed{V = A \times L}$

3) Stress $\sigma = \frac{F}{A}$

4) Modulus of resilience $= \frac{U(\text{strain energy})}{\text{Volume.}}$

Q. Calculate the strain energy store in a bar 2 m long. Having 50 mm width and 40 mm thickness. It subjected to a load 60 kN. Take E = 200 GPa

Data given :– $l = 2\,m = 2 \times 1000 = 2000\ mm$

$w = 50\ mm$

$t = 40\ mm$

$F = 60\ kN = 60 \times 10^3\ N$

$E = 200\ GPa = 200 \times 10^3\ N/mm^2$

Area $= w \times t \Rightarrow \omega = 50 \times 40 \Rightarrow w = 2000\ mm^2$

Stress $= \frac{F}{A} = \frac{60 \times 10^3}{2000} = 30\ N/mm^2$

Volume $= A \times \cancel{l} = 2000 \times 2000 = 4000000\ mm^3$

strain energy $= \frac{\text{strain energy (U)}}{\text{Volume}} = \frac{\sigma^2}{2E} \times V$

$= \frac{30^2}{2 \times 2 \times 10^5} \times 4 \times 10^6$

$= 9000\ N.m.$

(Case-2)

strain energy stored in a body when it is subjected to suddenly applied load.

1) Stress $(\sigma) = \frac{2F}{A}$

2) Volume (V) = Area x Length $\boxed{V = A \times L}$

3) Strain energy $(U) = \frac{\sigma^2}{2E} \times V$

4) Modulus of resilience $= \frac{U}{V}$

Q. A steel rod having 2.5m length and 1000 mm² area is subjected a suddenly applied load of 20 KN. Calculate the strain energy stored in the body. Take E = 200GPa

Data given:- length $(L) = 2.5\,m = 2.5 \times 1000$
$= 2500\,mm.$

Area $= 1000\,mm^2$

Force $(F) = 20\,KN = 20 \times 10^3\,N$

$E = 200\,GPa = 200 \times 10^3\,N/mm^2$

a) Stress $(\sigma) = \frac{2F}{A} = \frac{2 \times 200 \times 10^3}{1000} = 400\,N/mm^2$

b) Volume $= A \times L = 1000 \times 2500 = 2500000$
$= 25 \times 10^5\,mm^3$

c) $U = \frac{\sigma^2}{2E} \times V = \frac{(400)^2}{2 \times 2 \times 10^5} \times 25 \times 10^5$

$= 10^6\,N/mm$

Q. A steel rod of 15 mm diameter is subjected to a force of 10 KN. Calculate the strain energy stored in the body if the length of the rod 1 m. Take E = 200 GPa

Data given
$D = 15\ mm$
$F = 10\ KN = 10 \times 10^3\ N$
$L = 1\ m = 1000\ mm.$
$Area = \frac{\pi}{4} \times (15)^2 = 176.714$
$E = 200\ GPa = 200 \times 10^3\ N/mm^2$

$$\sigma = \frac{F}{A}$$

$$= \frac{10 \times 10^3}{176.714} = 56.58\ N/mm^2$$

Volume = Area x Length

$$= \cancel{56.58}\ 176.71 \times 1000 = 176710\ mm^3$$

$$= \cancel{56580\ mm^3}$$

$$U = \frac{\sigma^2}{2E} \times V$$

$$= \frac{(56.58)^2}{2 \times 200 \times 10^3} \times 176710$$

$$= 1414.25\ N.mm$$

Q. A steel rectangular column having 20 mm width and 10 mm thickness is subjected to a suddenly applied load of 1000 N. Calculate the strain Energy stored in the column if the length of the column is 2 m. Take E = 200 GPa

Given data:-
$W = 20\ mm.$
$t = 10\ mm$
$F = 1000 N$
$L = 2 \times 1000 = 2000\ mm$
$E = 200\ GPa = 2 \times 10^5\ N/mm^2$

Area = ~~20x10~~ = width x thickness

$$= 20 \times 10 = 200\ mm^2$$

Volume = Area x Length

$$= 200 \times 2000$$

$$= 400000\ mm^3$$

$$\sigma = \frac{2F}{A} = \frac{2 \times 1000}{200} = 10\ N/mm^2$$

$$\text{strain Energy} = \frac{\sigma^2}{2E} \times V$$

$$= \frac{(10)^2}{2 \times 2 \times 10^5} \times 400000$$

$$= \cancel{25\ N.mm}$$

$$= 100\ N.mm.$$

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Case 3

Strain energy stored in a body when a body subjected to impact load.

(1) Area $= \frac{\pi}{4}(d)^2$ or $w \times t$

(2) $\sigma = \frac{P}{A}\left[1 \pm \sqrt{1 + \frac{2AEh}{Pl}}\right]$ If h is less than 10 cm/100 mm

$\sigma = \sqrt{\frac{2EPh}{Al}}$ If h is more than or equal to 10 cm/100 mm.

Here, P = Force/Load

h = height

(3) $U = \frac{\sigma^2}{2E} \times V$

Q one more

A two meter long bar of 1500 mm² area is subjected to impact load of 2 kN, which falls from a height of 100 mm. Take E = 120 GPa and calculate the strain energy stored in the bar.

Data Given:- Length (l) = 2 m = 2×1000 = 2000 mm

F = 2 kN = 2×10^3 N

h = 100 mm.

E = 120 GPa = 12×10^4 N/mm²

A = 1500 mm²

~~Area~~ stress $(\sigma) = \sqrt{\frac{2EPh}{Al}}$

$= \sqrt{\frac{2 \times 12 \times 10^4 \times 2 \times 10^3 \times 100}{1500 \times 2000}}$

$= 126.49$ N/mm²

Volume (V) = Area × Length

$= 1500 \times 2000$

$= 3 \times 10^6$ mm³

Strain energy (U) $= \frac{\sigma^2}{2E} \times V$

$= \frac{(26.49)^2}{2 \times 12 \times 10^4} \times 3 \times 10^6$

$= 199996.50$ N.mm

Q

A steel bar 3 m long and 2500 mm² in area is subjected to a impact load of 15 kN which falls from a height of 10 mm. Calculate the strain energy stored in the body. Take E = 200 GPa.

Given data:- L = 3 m = 3×1000 = 3000 mm

A = 2500 mm²

F = 15 kN = 15×10^3 N

h = 10 mm.

E = 200 GPa = 200×10^3

$= 2 \times 10^5$ N/mm²

$$\sigma = \frac{P}{A}\left[1+\sqrt{1+\frac{2AEh}{PL}}\right]$$

$$= \frac{15\times10^3}{2500}\left[1+\sqrt{1+\frac{2\times2500\times2\times10^5\times10}{15\times10^3\times3000}}\right]$$

$$= 6\left[15.90\right]$$

$$= 95.4\ N/mm^2$$

V = Area × Length

$= 2500 \times 3000$

$= 75\times10^5\ mm^3$

$$U = \frac{\sigma^2}{2E}\times V$$

$$= \frac{(95.4)^2}{2\times2\times10^5}\times 75\times10^5$$

$= 170646.75$ N.mm.

$= 170.6\times10^3$ N.mm, (171.50×10^3 N.mm)

TYPES OF STRAIN

There are 2 types of strain

(1) Linear Strain

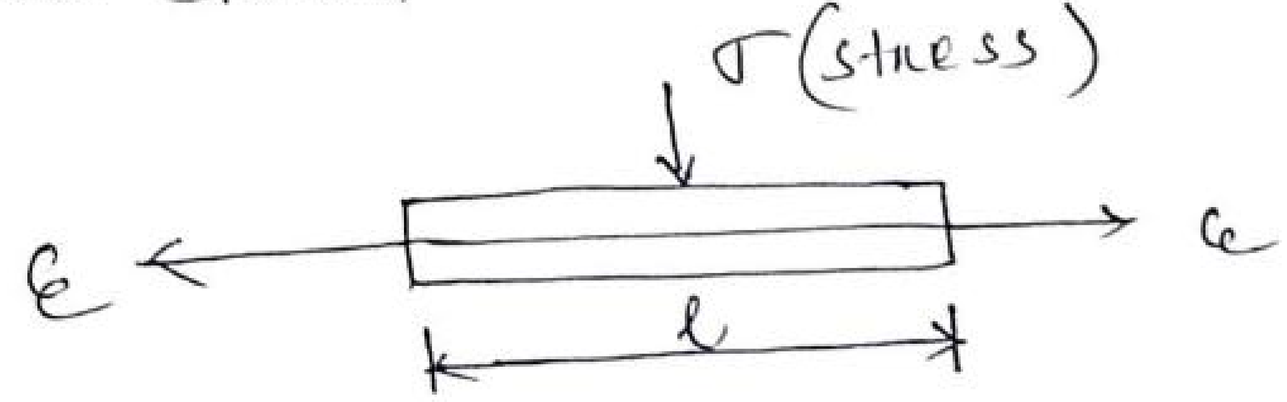
(2) Lateral Strain

1) LINEAR STRAIN :-

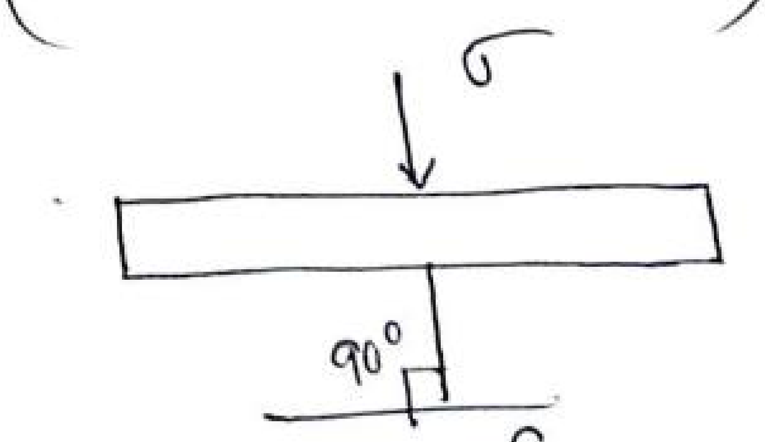
The strain which when acts on a body results in increase in length along its linear direction is called linear strain.

2) LATERAL STRAIN :-

The strain which is subjected on a body or act on a body at right angle is known as lateral strain.



(Linear Strain)



(Lateral Strain)

b = width

t = thickness

Lateral strain $= \frac{\Delta b}{b} = \frac{\Delta t}{t}$

Δb = change in width

Δt = change in thickness

Linear strain $= \frac{\Delta L}{L}$

POISSON'S RATIO:-

Possion's ratio can be defined as the ratio of lateral strain to linear strain.

* It is denoted by $\frac{1}{m}$ or μ

* $\boxed{\frac{1}{m} \text{ or } \mu = \frac{\text{Lateral strain}}{\text{Linear strain}}}$

Q. A steel bar 2 m long, 40 mm width and 20 mm thick is subjected to a force of 160KN. Calculate
i) ΔL (Change in Length)
ii) ~~width~~ (Δb) (Change in width)
~~iii) Thickness of bar (t)~~
iii) ~~etc~~ Δt (Chang in thickness)

Take E = 200 GPa and poisson's ratio is 0.1

$\Delta L = \frac{F \times L}{A \times E}$ lateral strain $\cancel{\delta b} = \frac{\Delta b}{b}$ lateral strain $\cancel{\delta t} = \frac{\Delta t}{t}$

Area = width x thickness
$= 40 \times 20 = 800\ mm^2$

$F = 160 KN = 160 \times 10^3 N$
$l = 2m = 2000\ mm$
$E = 200\ GPa = 2 \times 10^5\ N/mm^2$

$\Delta L = \frac{F \times L}{A \times E}$

$= \frac{160 \times 10^3 \times \cancel{2000}^{\,10}}{\cancel{800}_{4} \times \cancel{2} \times 10^5}$

$= 2\ mm$

$\mu = \frac{\Delta b}{b}$

$\Rightarrow \Delta b = \mu \times b = 0.1 \times 40 = 4\ mm$

$\mu = \frac{\Delta t}{t}$

$\Rightarrow \Delta t = \mu \times t = 0.1 \times 20 = 2\ mm$

Q. A metal bar 50x50 mm is subjected a load of 500KN. If the total length of 200mm has an elongation of 0.5 mm and the increase in thickness is 0.04 mm. Calculate
(i) E (young's modulos)
(ii) Poission's ratio.
(iii) Change in width.

Data given
$A = 50 \times 50 = 2500\ mm^2$
$F = 500\ KN = 5 \times 10^5\ N$
$\Delta l = 0.5\ mm$
$l = 200\ mm$
$\Delta t = 0.04\ mm.$
$b = 50\ mm.$
$t = 50\ mm$

(i) $\Delta L = \frac{F \times L}{A \times E}$

$\Rightarrow E = \frac{F \times l}{A \times \Delta l} = \frac{5 \times 10^5 \times 200}{\cancel{2500}_{5} \times 0.5}$

$= 80000 = 8 \times 10^4$

(ii) Linear strain $= \frac{\Delta l}{l} = \frac{0.5}{200} = 2.5 \times 10^{-3}$

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Lateral strain $= \frac{\Delta t}{t}$

$= \frac{0.04}{50} = 8\times10^{-4}$

Poisson's ratio $= \frac{\text{Lateral strain}}{\text{Linear strain}}$

$= \frac{8\times10^{-4}}{2.5\times10^{-3}}$

$= 0.32\ (\mu)$

(iii) ~~Change in width Δb~~

~~$\Rightarrow \mu = \frac{\Delta b}{b}$~~

~~$\Rightarrow 0.32 \times = \frac{\Delta b}{b}$~~

~~$\Rightarrow \Delta b = 0.32\times50 = 16$ mm.~~

(iv) Change in width (Δb)

Lateral strain $= \frac{\Delta b}{b}$

$\Rightarrow \Delta b =$ Lateral strain $\times\ b$

$\Rightarrow \Delta b = 8\times10^{-4}\times50 = 0.04$ mm.

Q. A 2m long steel bar 40×20 mm is subjected to a load 250 KN. Calculate

i) Young's Modulus.

ii) Poisson's ratio

iii) Change in thickness.

Take ~~E = 200~~ , $\Delta l = 0.6$ mm, $\Delta b = 0.05$ mm

Given data :- $l = 2\text{ m} = 2\times1000 = 2000$ mm.

$A = 40\times20 = 800\text{ mm}^2$

$F = 250\text{ KN} = 25\times10^4$ N

$w = 40$ mm.

$t = 20$ mm.

$\Delta l = 0.6$ mm

$\Delta b = 0.05$ mm.

(i) $E = \frac{F\times L}{A\times\Delta l}$

$= \frac{25\times10^4\times\cancel{2000}^{10^5}}{\cancel{800}_{4\cdot2}\times0.6} = 1041666.667\text{ N/mm}^2$

(ii) Linear strain $= \frac{\Delta l}{l} = \frac{0.6}{2000} = 3\times10^{-4}$

Lateral strain $= \frac{\Delta b}{b} = \frac{0.05}{40} = 1.25\times10^{-3}$

Poisson's ratio $= \frac{\text{Lateral strain}}{\text{Linear strain.}}$

$= \frac{1.25\times10^{-3}}{3\times10^{-4}} = 4.16.$

(iii) Change in thickness (Δt)

Lateral strain $= \frac{\Delta t}{t}$

$\Rightarrow \Delta t =$ Lateral strain $\times\ t$

$= 1.25\times10^{-3}\times20$

$= 0.025$ mm.

BULK MODULUS (K)

(i) Bulk modulus can be defined as the ratio of stress to the volumetric strain.

(ii) Bulk modulus $= \dfrac{\text{Stress}}{\text{Volumetric strain}} =$ N/m²

$$\boxed{K = \dfrac{\sigma}{\frac{\Delta V}{V}}}$$

* Volumetric strain $= \dfrac{\Delta V}{V}$

(iii) Volumetric strain can be defined as the ratio of change in volume to the original volume.

Q. Calculate the bulk modulus for a material having 400000 m³ volume and change in volume is equal to 0.05. The material is subjected to a force of 100 N having diameter of 45 mm.

Given data $V = 400000\ m^3$

$\Delta V = 0.05\ m^3$

$F = 100\ N$

$d = 45\ mm$

$= 0.045\ m$

Area $= \frac{\pi}{4}(d)^2$

$= \frac{\pi}{4} \times (0.045)^2$

$= 1.59 \times 10^{-3}\ m^2$

$\sigma = \dfrac{F}{A} = \dfrac{100}{1.59 \times 10^{-3}}$

$= 62893.08\ N/m^2$

$K = \dfrac{\sigma}{\frac{\Delta V}{V}}$

$= \dfrac{62893.08}{\frac{0.05}{400000}}$

$= 5.03 \times 10^{110}\ N/m^2$

$$\boxed{K = 5.03 \times 10^{11}\ N/m^2}$$

Q. A steel rod having 50 mm diameter is subjected to a load of 10 KN. Calculate the change in volume for the material having volume 35000 m³ and bulk modulus (K) $47 \times 10^5\ N/m^2$

Data given = $D = 50\ mm = 0.050\ m$

$F = 10\ KN = 10^4\ N$

$V = 35000\ m^3$

$K = 47 \times 10^{-5}\ N/mm^2$ $\quad \Delta V = ?$

$$\text{Area} = \frac{\pi}{4}(d)^2$$
$$= \frac{\pi}{4}(0.05)^2$$
$$= 1.96 \times 10^{-3}\ m^2$$

$$\sigma = \frac{F}{A} = \frac{10^4}{1.96\times10^{-3}}$$
$$= 5102040.81\ N/m^2$$

$$K = \frac{\sigma}{\frac{\Delta V}{V}}$$

$$\Rightarrow 47\times10^5 = \frac{5102040.81}{\frac{\Delta V}{35000}}$$

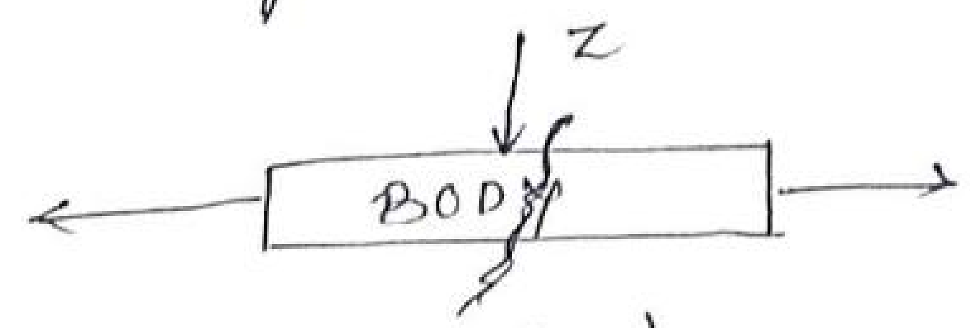
$$\Rightarrow 47\times10^5 = \frac{5102040.81\times35000}{\Delta V}$$

$$\Rightarrow \Delta V = \frac{5102040.81\times35000}{47\times10^5}$$
$$= 37993.92\ m^3$$

MODULUS OF RIGIDITY (G/C)

SHEAR STRESS :- (τ) (Tow)

Shear stress can be defined as the couple of ~~forces~~ equal and opposite forces, which when subjected to a body results in breaking of the body into different section.



SHEAR STRAIN :- (ϕ)

Shear strain can be defined as the strain produced in the body due to shear stress.

M.O.R :- (G/C) → It can be defined as the ratio of shear stress to shear strain.

$$G/C = \frac{\text{Shear stress}}{\text{shear strain}} = \frac{\tau}{\phi} = (N/mm^2)$$

ELASTIC CONSTANTS

(i) Elastic constant are the constant which are used to measure the strength of a particular material.

(ii) There are 3 type of elastic constant.

(1) Young's modulus (E)
(2) Bulk modulus (K)
(3) Modulus rigidity (G/C)

(1) $E = \frac{\sigma}{\epsilon}$ (2) $K = \frac{\sigma}{\frac{\Delta V}{V}}$ (3) $g = \frac{\tau}{\phi}$

— o —

(CH-2) **THIN CYLINDER**

* The cylinders having thickness less than $\frac{1}{10}$th to $\frac{1}{15}$th of the diameter are known as thin cylinders.
* The cylinders having thickness greater than $\frac{1}{10}$th to $\frac{1}{15}$th of diameter are known as thick cylinder.

$t = 10$ $d = 500$

$\frac{500}{10} = 50$ $\frac{500}{15} = 33.33$

$10 < 50$ $10 < 33.33$ (Thin cylinder)

$60 > 50$ $60 > 33.33$ (Thick cylinder)

INTERNAL PRESSURE INSIDE CYLINDER

When cylinders are subjected to internal pressure, this pressure exerts force on the walls on the cylinder.

If the pressure inside the cylinder exceeds the maximum value than it results in the failure of the cylinder

FAILURE OF PRESSURE VESSELS

Whenever the pressure inside the pressure vessels exceeds the maximum value than failure of the pressure vessels takes place.

These failure takes place under the action of two stresses. They are

i) Circumferential stress

ii) Longitudinal stress.

i) **CIRCUMFERENTIAL STRESS**:— (Hoop stress)

* The stress which is subjected along the circumference on the diameter of the cylinder is known as circumferential stress.
* It is denoted as σ_c
* Circumferential stress can be calculated by using formula

$$\boxed{\sigma_c = \frac{Pd}{2t}}$$

Where σ_c = Circumferential stress

P = Internal pressure

d = diameter of cylinder

t = thickness of cylinder plate.

* When efficiency of the joint given in that case circumferential stress can be calculated by using formula.

$$\boxed{\sigma_c = \frac{pd}{2t\eta}}$$ η = efficiency given

ii) LONGITUDINAL STRESS :-

* When the stress is subjected along the length of the cylinder, this stress is known as Longitudinal stress.
* It is denoted by σ_l.
* Longitudinal stress can be calculated by using formula

$$\boxed{\sigma_l = \frac{pd}{4t}}$$

σ_l = Longitudinal stress
p = internal pressure
d = dia of cylinder
t = thickness of cylinder.

* If the efficiency of the joint is given then

$$\boxed{\sigma_l = \frac{pd}{4t\eta}}$$ η = efficiency

Q A cylinder is subjected to an internal pressure of 2 MPa having diameter 40 mm and thickness 10 mm. calculate the hoop's stress / circumferential stress for the given cylinder.

Given data :- $P = 2\text{ MPa} = 2\text{ N/mm}^2$
$d = 40\text{ mm}$
$t = 10\text{ mm}$

$$\sigma_c = \frac{pd}{2t} = \frac{2 \times \overset{4}{\cancel{40}}}{\cancel{2} \times \cancel{10}} = 4\text{ N/mm}^2$$

$$\boxed{\sigma_c = 4\text{ N/mm}^2}$$

Q A thin cylinder is subjected to an internal pressure of 8 MPa having diameter 25 mm and thickness 12 mm. Calculate the longitudinal stress.

Given data :- $P = 8\text{ MPa} = 8\text{ N/mm}^2$
$d = 25\text{ mm}$
$t = 12\text{ mm}$

$$\sigma_l = \frac{pd}{4t} = \frac{\cancel{8} \times 25}{\cancel{12} \times 4} = \frac{25}{\cancel{6}\,3} = 4.16\text{ N/mm}^2$$

$$\boxed{\sigma_l = 4.16\text{ N/mm}^2}$$

Q calculate the circumferential stress and longitudinal stress for a cylinder having 30 mm dia and 8 mm thickness of plates subjected to an internal pressure of 6.5 MPa. Take the efficiency of the joint as 75%.

Given data :- $P = 6.5\text{ MPa} = 6.5\text{ N/mm}^2$
$d = 30\text{ mm}$ $\eta = 75\%$
$t = 8\text{ mm}$

$$\sigma_c = \frac{Pd}{2t\eta}$$

$$= \frac{6.5 \times \cancel{30}^{15}}{\cancel{2} \times 8 \times 0.75} = 16.25\ N/mm^2$$

$$\sigma_l = \frac{Pd}{4t\eta}$$

$$= \frac{6.5 \times 30}{4 \times 8 \times 0.75} = 8.125\ N/mm^2$$

DESIGN OF SPHERICAL SHEEL

Here we have to calculate the thickness of the spherical shell

$$\boxed{t = \frac{Pd}{2\sigma_c}}$$

where P = internal Pressure.
t = thickness of Spherical shell
d = dia of spherical
σ_c = circumferential stress

If efficiency is given

$$\boxed{t = \frac{Pd}{2\sigma_c \eta}}$$

η = efficiency.

Q A thin cylindrical shell of 400mm dia is subjected to an internal pressure of 2.4 MPa. calculate the thickness of the shell if circumferential stress is 50 MPa.

Data given :- d = 400mm.
P = 2.4 MPa
= 2.4 N/mm²
σ_c = 50 MPa
= 50 N/mm²

$$t = \frac{Pd}{2\sigma_c} = \frac{2.4 \times \cancel{400}^{8}}{\cancel{50}} = 9.6\ mm.$$

Q A shell of 500mm diameter is subjected to an internal pressure of 4 MPa. Calculate the thickness of the shell if ultimate circumferential stress is given 400 MPa. Take efficiency of the joint equals to 65%. and factor of safety equals to 5

Data given
d = 500mm
P = 4 MPa = 4 N/mm²
ul σ_c = 400 MPa = 400 N/mm²
η = 65% = 0.65
FOS = 5

$$\boxed{\text{circumferential stress} = \frac{ul\,\sigma_c}{FOS}}$$

$$= \frac{400}{5} = 80\ N/mm^2$$

$$t = \frac{Pd}{2\sigma_c \eta} = \frac{4 \times \cancel{500}^{25}}{2 \times \cancel{80}_{8} \times .65} =$$

$$= 19.25\ mm$$

CHANGES IN DIMENSION OF THIN CYLINDRICAL SHELL :-

1) changes in diameter (Δd)

$$\boxed{\Delta d = \frac{Pd^2}{2tE}\left(1-\frac{1}{2m}\right)}$$

P = internal Pressure
d = diameter
t = thickness
E = Young's Modulus
$\frac{1}{m}$ = Poision ratio

or $\left(1-\frac{1}{2m}\right) = \left(1-\frac{1}{2}\times\frac{1}{m}\right)$

2) Change in length (Δl)

$$\boxed{\Delta L = \frac{Pdl}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)}$$

l = Length of the cylinder.

Q A cylindrical thin drum 800mm in dia and 4m long is made of 10mm thick plates. If the drum is subjected to an internal pressure of 2.5 MPa. Calculate change in dia and change in length. Take E = 200 GPa and poisons ratio = 0.25

~~Data~~ 0

Data given . $d = 800\,mm$

$l = 4m = 4000\,mm$

$t = 10\,mm$

$P = 2.5\,MPa = 2.5\,N/mm^2$

$E = 200\,GPa = 2\times10^5\,N/mm^2$

$\frac{1}{m} = 0.25$

$$\Delta d = \frac{Pd^2}{2tE}\left(1-\frac{1}{2m}\right)$$

$$= \frac{2.5\times(800)^2}{2\times10\times2\times10^5}\left(1-\frac{1}{2}\times0.25\right)$$

$$= 0.35\,mm$$

$$\Delta l = \frac{Pdl}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)$$

$$= \frac{2.5\times800\times4000}{2\times10\times2\times10^5}\left(\frac{1}{2}-0.25\right)$$

$$= 0.5\,mm$$

CIRCUMFERENTIAL STRAIN & LONGITUDENAL STRAIN :-

We know that

$$\Delta d = \frac{Pd^2}{2tE}\left(1-\frac{1}{2m}\right)$$

$$\Rightarrow \frac{\Delta d}{d} = \frac{Pd}{2tE}\left(1-\frac{1}{2m}\right)$$

$$\Rightarrow \boxed{\varepsilon_c = \frac{Pd}{2tE}\left(1-\frac{1}{2m}\right)}$$

ε_c = circumferential strain

we know that

$$\Delta l = \frac{Pdl}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)$$

$$\Rightarrow \frac{\Delta l}{l} = \frac{Pd}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)$$

$$\Rightarrow \boxed{\varepsilon_L = \frac{Pd}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)}$$

ε_l = longitudenal Strain.

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3) Change in Volume (ΔV)

$$\Delta V = V(\varepsilon_l + 2\varepsilon_c)$$

where V = Volume = $A \times l = \frac{\pi}{4} \times (d)^2 \times l$

ε_l = Longitudinal strain

ε_c = Circumferential strain.

Q A cylindrical vessels 2m long at 500mm in dia with 10 mm thick plates is subjected to an internal pressure of 3MPa. Calculate

i) Circumferential strain

ii) Longitudinal strain

iii) Change in diameter (ΔD)

iv) Change in length

v) Volume

vi) Change in volume.

Take $E = 200GPa$ (2×10^5) & $\frac{1}{m} = 0.3$

Given data $l = 2m = 2000\,mm$

$d = 500\,mm$

$t = 10\,mm$

$P = 3MPa = 3\,N/mm^2$

i) $\varepsilon_c = \frac{Pd}{2tE}\left(1 - \frac{1}{2m}\right)$

$= \frac{3 \times 500}{2 \times 10 \times 2 \times 10^5}\left(1 - \frac{1}{2} \times 0.3\right)$

$= 3.1875 \times 10^{-4}$

ii) $\varepsilon_l = \frac{Pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$

$= \frac{3 \times 500}{2 \times 10 \times 2 \times 10^5}\left(\frac{1}{2} - 0.3\right)$

~~= 0.0375 × 0.2~~

$= 7.5 \times 10^{-5}$

iii) $\Delta d = \varepsilon_c \times d$

$= 3.1875 \times 10^{-4} \times 500$

$= 0.159\,mm$

iv) $\Delta l = \varepsilon_l \times l$

$= 7.5 \times 10^{-5} \times 2000$

$= 0.15\,mm$

v) V = Area × length

$= \frac{\pi}{4}(d)^2 \times \text{length}$

$= \frac{\pi}{4} \times (500)^2 \times 2000$

$= 392699081.7\,mm^3$

vi) $\Delta V = V(\varepsilon_l + 2\varepsilon_c)$

$= \cancel{392699081.7 + 2 \times 3.1875 \times 10^{-4}}$

$= 392699081.7(7.5 \times 10^{-5} + 2 \times 3.1875 \times 10^{-4})$

$= 279209.04\,mm^3$

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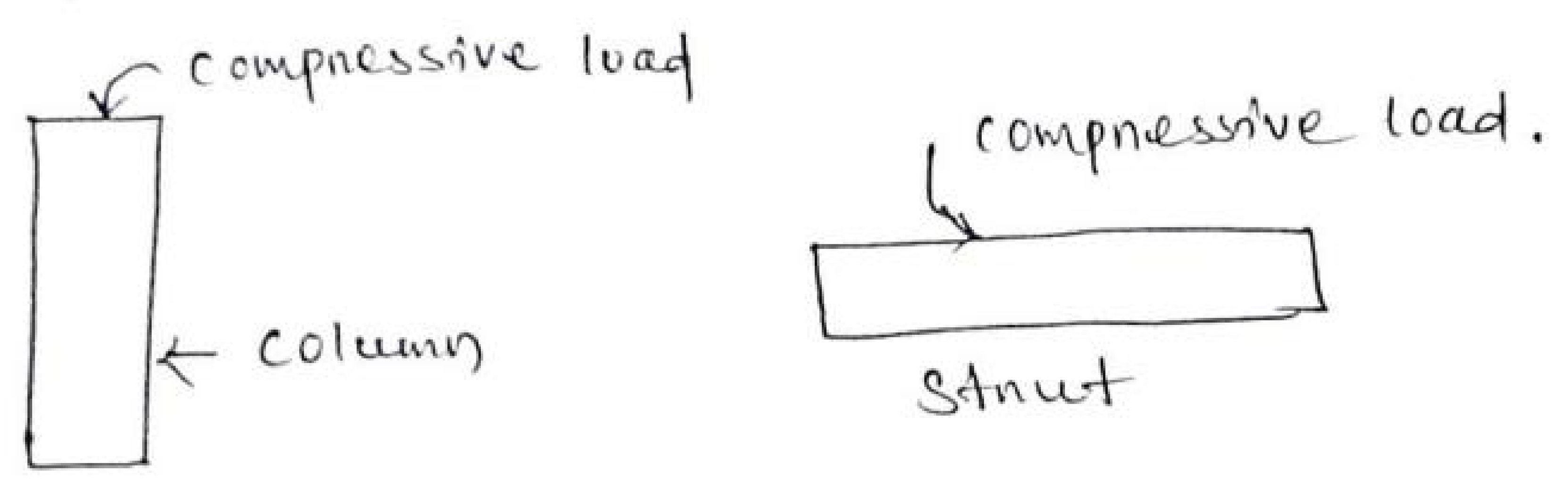
(CH-3) COLUMNS

A column can be defined as any structural member subjected to axial compressive force. Strut a horizontal structural member subjected to compressive force is known as strut.

In other words a column is a vertical strut.



FAILURES OF A COLUMN:—

There are two types of columns

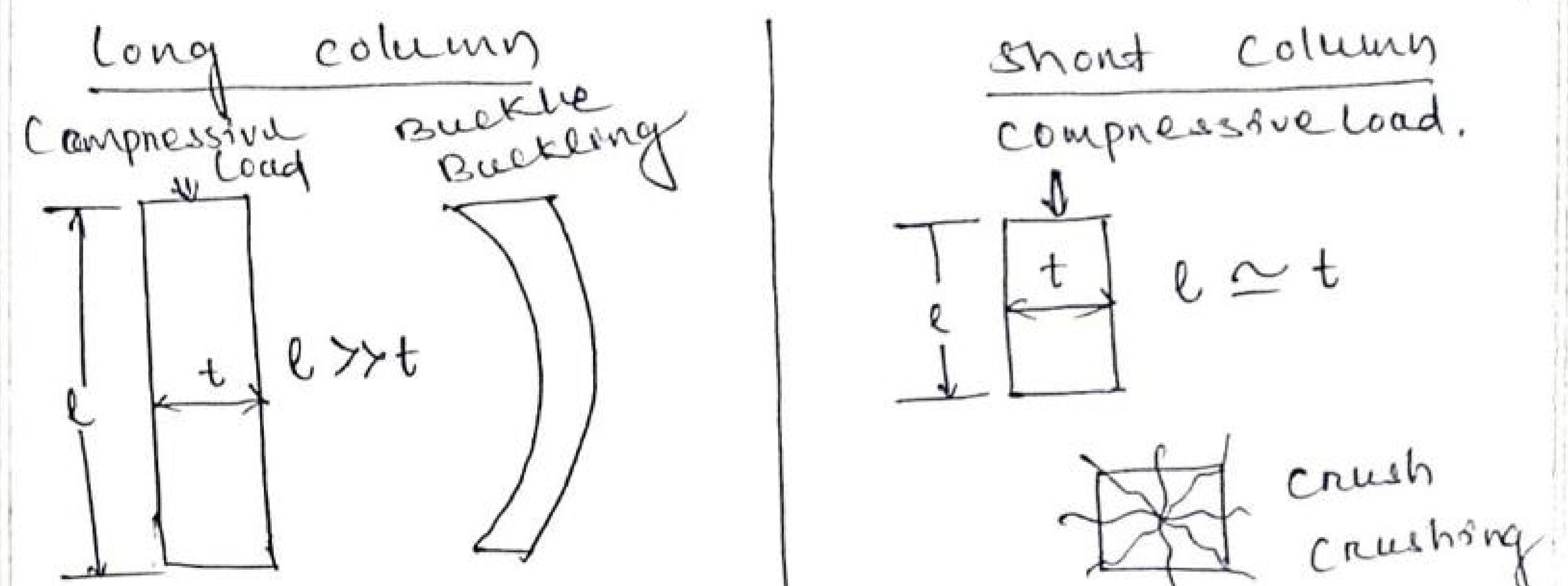
i) Long column
ii) Short column

i) **Long column**:— In case of long columns the compressive force subjected on the column results in bending of the column. This phenomena is known as buckling of column.

Buckling load can be defined as the compressive load responsible for the buckling of column is known as buckling load. It is also known as Crippling load.

ii) **Short column**:— In case of short columns the length of the column is nearly equal to the thickness of the column hence bending of the column is not possible.

But when the compressive load subjected on the column exceeds the maximum value then the short column fail due to ~~crossing~~ Crushing.



EULER'S THEORY OF COLUMNS:— (Applicable for long column)

Euler's theory of column is used to calculate the buckling load subjected on a long column.

ASSUMPTION OF E.T.C:— I.M.P

i) The column should be perfectly straight and the load should be axial.
ii) The column should only fail due to buckling.
iii) The column should be always subjected to compressive load.
iv) The length of the column must be large as compared to the other dimension.

v) The cross sectional area of the column should be uniform throughout its length.

vi) The column should be perfectly elastic, homogeonous & Isotropic → throughout the column same all parts. (all properties same)

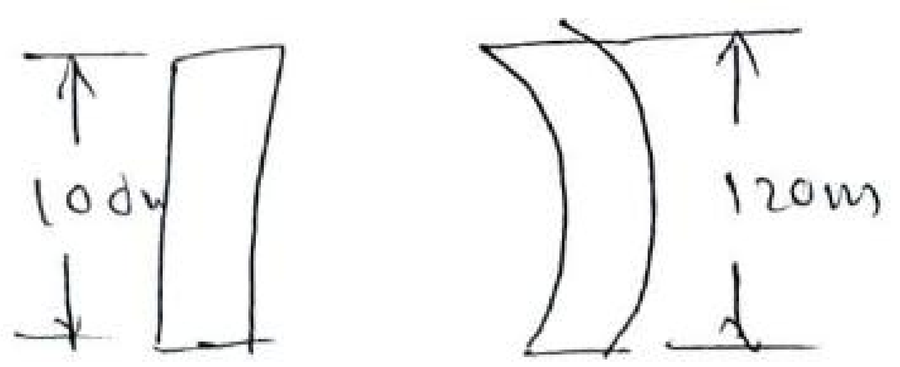
END CONDITION OF A COLUMN SUBJECTED TO EULER'S THEORY OF COLUMN :-

There are 4 end condition of columns subjected to uler's theory. They are

i) Both the ends are hinged & pinned.
ii) Both the ends are fixed
iii) One end is fixed & other end is hinged
iv) One end is fixed & other end is free

EQUIVALENT LENGTH OF COLUMN :-

When long columns are subjected to a buckling load then the column ~~is~~ subjected to the process of buckling. The gradual length of the column after buckling is known as equivalent length of the column. on effective of the column.



It is denoted by the symbol L_e

FORMULA ~~FOR~~ TO CALCULATE BUCKLING LOAD OR CRIPPLING LOAD USING EULER'S THEORY

$$\boxed{P = \frac{\pi^2 E I}{L_e^2}}$$

where P = buckling load on crippling load
E = Young's Modulus
I = Moment of Inertia
Le = equivalent Length of Column.

RELATION BETWEEN EQ. LENGTH OF COLUMN & LENGTH OF THE COLUMN IN DIFFERENT END CONDITION :-

End condition	Relation betw Le & L
1) Both ends are hinged & pinned →	$L_e = l$
2) Both ends are fixed →	$L_e = \frac{l}{2}$
3) One end is fixed and other is hinged →	$L_e = \frac{l}{\sqrt{2}}$
4) One end is fixed and other is free →	$L_e = 2l$

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1) Moment of inertia for circular section (solid)

$= \frac{\pi}{64}(d)^4$

2) circular section (hollow)

$= \frac{\pi}{64}\left[D^4 - d^4\right]$

3) Rectangular section (solid)

$= \frac{bd^3}{12}$

4) Rectangular section (hollow)

$= \frac{1}{12}\left[BD^3 - bd^3\right]$

Q. A steel rod 5m long and 40 mm in diameter is used as a column ~~width~~ with one end is fixed and other end free. Calculate the crippling load using uler's formula. Take E = 200 GPa

Data given : $l = 5m = 5000 mm$.
$d = 40 mm$
$E = 200 GPa = 2\times10^5 N/mm^2$

$P = \frac{\pi^2 EI}{Le^2}$

For one end fixed and another is free

so $Le = 2l$
$= 2\times5000$
$= 10000 mm$
$= 10^4 mm$.

Moment of inertia of cylinder (solid)

$I = \frac{\pi}{64}(40)^4$
$= 125663.70 \ mm^4$

$P = \frac{\pi^2 EI}{Le^2}$

$= \frac{\pi^2 \times 2\times10^5 \times 125663.7}{(10^4)^2}$

$= 2480.5 N$

Q. A rectangular column having 200mm width and 100mm thickness is ~~subjec~~ fixed at one end and hinged at the other end. Calculate the crippling load on the column using uler's theory if the length of the column is 6mo Take E = 200 GPa

Given data : $l = 6m = 6000 mm$.
$b = 200 mm$.
(t) $d = 100 mm$
$E = 200 GPa = 2\times10^5 N/mm^2$

for one end fixed and other hinged

$Le = \frac{l}{\sqrt{2}} = \frac{6000}{\sqrt{2}} = 4242.64 mm$.

for rectangular column

$I = \frac{bd^3}{12} = \frac{200\times100^3}{12}$
$= 16666666.67 \ mm^3$

$$P = \frac{\pi^2 EI}{Le^2}$$

$$= \frac{\pi^2 \times 2\times10^5 \times 1.66666667}{(6000)^2}$$

Q A hollow tube 4m long with external and internal diameter of 40mm and 25 mm respectively was found to be extended by 4.8 mm under a load of 60KN. Calculate the crippling load if the both the ends are hinged

Data given :- $D = 40mm$ $d = 25mm$

$F = 60KN = 6\times10^4 N$ $l = 4m = 4000mm$

$\Delta L = 4.8mm$.

both the ends are hinged

$Le = l$

$Le = 4000$

$$Area = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}((40)^2 - (25)^2) = \frac{\pi}{4}(1600 - 625)$$

$$= 765.76\ mm^2$$

$$\sigma = \frac{F}{A} = \frac{6\times10^4}{765.76} = 78.35 N$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{4.8}{4000} = 1.2\times10^{-3}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{78.35}{1.2\times10^{-3}} = 65291.66 N/mm^2$$

$$\therefore P = \frac{\pi^2 EI}{Le^2}$$

$$= \frac{\pi^2 \times 65291.66 \times 106488.94}{4000^2}$$

$$= 4288.86 N \text{ (Ans)}$$

Q A rectangular column having 250mm width and 100 mm thickness is 5m long and subjected to a load of 50KN. Calculate the buckling load if both the ends are fixed and having a change in length of 4.5 mm

Given data : ⇒ $w = 250mm$, $t = 100mm$

$l = 5m = 5000mm$, $\Delta L = 4.5mm$

$F = 50KN = 5\times10^4 N$

both the ends are fixed so

i) $Le = \frac{L}{2} = \frac{5000}{2} = 2500mm.$

ii) $I = \frac{bd^3}{12} = \frac{250\times(100)^3}{12} = 20833333.33$

iii) $A = w\times t = 250\times100 = 25000\ mm^2$

iv) $E = \frac{\sigma}{\varepsilon}$

$$\sigma = \frac{F}{A} = \frac{5\times10^4}{25000} = 2\ N/mm^2$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{4.5}{5000} = 9\times10^{-4}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{2}{9\times10^{-4}} = 2222.22\ N/mm^2$$

v) $F = \frac{\pi^2 E I}{Le^2}$

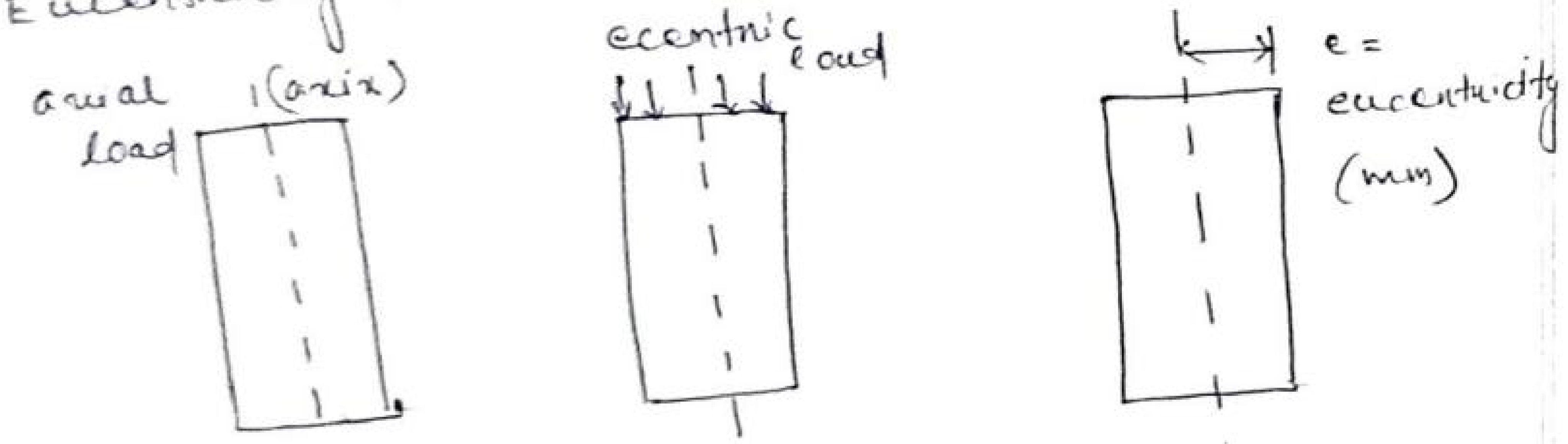
$= \frac{\pi^2 \times 2222.22 \times 20833333.33}{2500^2}$

$= 73108.10\ N$

ECCENTRIC LOADING IN COLUMNS :-

~~Eccentric load~~ Eccentric load :- The load which donot coincide with the axis of the column is known a eccentric load.

~~Ecc~~ (e) ~~Eccentricity~~ Eccentricity :- The distance between axis of the column and eccentric load is known as "Eccentricity".



COLUMNS SUBJECTED TO DIRECT & BENDING STRESSES :-

Let P = Load acting on the column.
b = width of the column.
t = thickness of the column.
e = Eccentricity

Case 1

When width of the column is given

i) Maximum intensity of stress (σ_{max}) $= \frac{P}{A}\left(1 + \frac{6e}{b}\right)$

ii) Minimum intensity of stress (σ_{min} $\frac{P}{A}\left(1 - \frac{6e}{b}\right)$

Case - 2

When width of the column is not given on section modulus is provided.

i) Max intensity of stress (σ_{max}) $= \frac{P}{A} + \frac{M}{Z}$

ii) Min intensity of stress (σ_{min}) $= \frac{P}{A} - \frac{M}{Z}$

where,

P = force acting on column.
A = Cross sectional area
M = Bending moment ($P \times e$)
Z = Section Modulus ($\frac{\pi}{32} \times d^3$)

SECTION MODULUS (Z) :-

Section modulus can be defined as the force acting on a particular body which do not coincide with the axis of the body and brings a bending moment is knowne as section modulus (Z)

The section modulus can be calculated for the formula :-

i) For rectangular section (solid) $\Rightarrow Z = \frac{Pd^2}{6}$ (mm^3)

ii) For rectangular section (hollow) $\Rightarrow Z = \frac{bd^2}{6}$ (mm^3)

iii) For circular section (solid) $\Rightarrow Z = \frac{\pi}{32} \times d^3$

iv) For circular section (hollow) $\Rightarrow Z = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)$

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Q. A rectangular column 150X120mm it carries a load 180 KN at an eccentricity of 10mm. Calculate the maximum and minimum intensity of stress.

Given data :- $b = 1500$ mm $d = 120$ mm,
$e = 10$ mm $P = 180KN = 18\times10^4$ N

Area $= b\times d = 150\times120 = \cancel{18000}\ 180\times10^3$

Case 1 :-

$$\sigma_{max} = \frac{P}{A}\left(1+\frac{6e}{b}\right)$$

$$= \frac{18\times10^4}{18\times10^3}\left(1+\frac{6\times10}{\frac{120}{2}}\right) = 14 \text{ Pa}$$

$$\sigma_{min} = \frac{P}{A}\left(1-\frac{6e}{b}\right)$$

$$= \frac{18\times10^4}{18\times10^3}\left(1-\frac{6\times10}{120}\right) = 6 \text{ Pa}$$

Q. A circular column having 200mm diameter is subjected to a load 185KN with an eccentricity of 70mm. Calculate the max and min intensity of load.

Given data :- $d = 200$ mm.
$F = 185\ KN = 185\times10^3$ N
$e = 70$ mm

$$\sigma_{max} = \frac{P}{A}+\frac{M}{Z}, \quad \sigma_{min} = \frac{P}{A}-\frac{M}{Z}$$

$M = P\times e$
$= 185\times10^3\times70$
$= 1295\times10^4$

$A = \frac{\pi}{4}d^2$
$= \frac{\pi}{4}\times200^2$
$= 31415.92$ mm^2

$$\sigma_{max} = \frac{185\times10^3}{31415.92} + \frac{1295\times10^4}{785398.16}$$

$= 22.377$ Pa

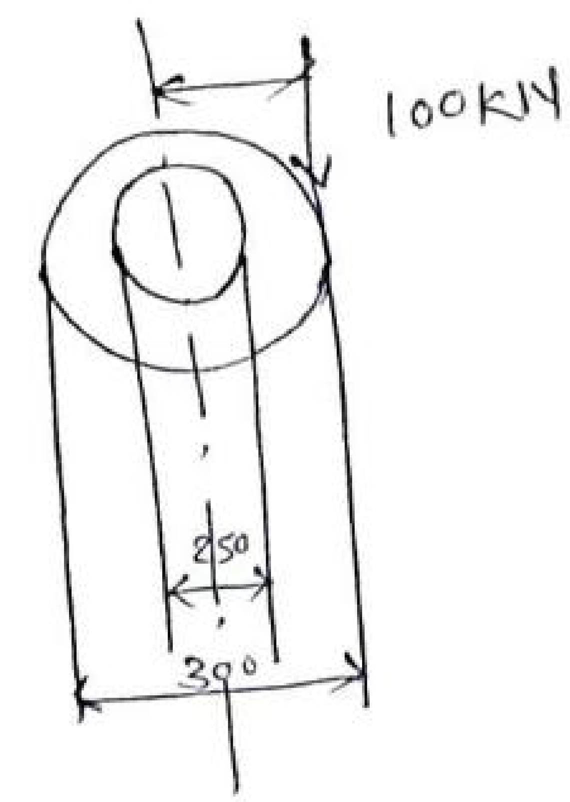
$$\sigma_{min} = \frac{185\times10^3}{31415.92} - \frac{1295\times10^4}{785398.16}$$

$= -10.599$ Pa.

$Z = \frac{\pi}{32}\times d^3$
$= \frac{\pi}{32}(200)^3$
$= 785398.16$

Q. A hollow circular column having external diameter of 300 mm and internal diameter of 250 mm respectively carries a vertical load of 100 KN at the edge of the column. Calculate the max and min intensity of load.

Given data :- $D = 300$ mm
$d = 250$ mm
$P = 100KN = 100\times10^3 = 10^5$ N



$e = \frac{300}{2} = 150$ mm

$M = P\times e = 10^5\times150$
$= 15\times10^6$

$$Z = \frac{\pi}{32}\left[\frac{D^4-d^4}{D}\right]$$

$$= \frac{\pi}{32}\left[\frac{300^4-250^4}{300}\right]$$

$= \cancel{137240}$

$= 1372401.47$

$A = \frac{\pi}{4}(D^2-d^2)$

$= \frac{\pi}{4}(300^2-250^2)$

$= 21598.44$ mm^2

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(CH-4) (TORSION)

SHAFT :- shaft can be defined as a rotating machine element which transmit power any motion.

TURNING FORCE :- The force applied on a shaft which results in twisting of that shaft is known as turning force.

TORSION / TORQUE :-

The product of turning force and the distance from the point of application to the axis of the shaft is known as turning moment. When the shaft is subjected to turning moment it is set to be in torsion / torque.

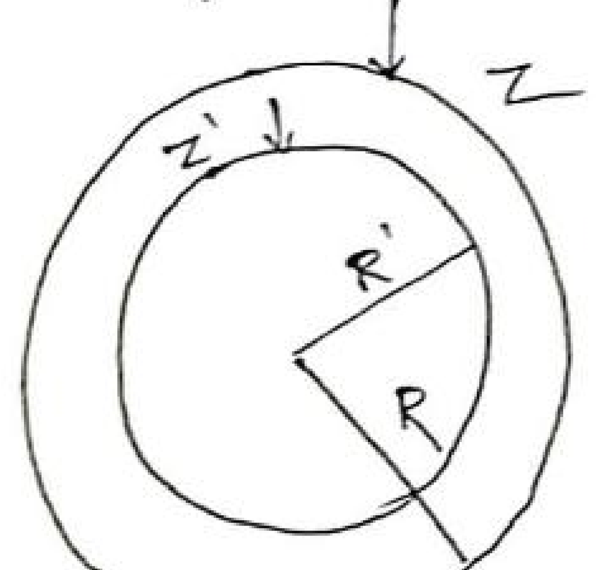
$\boxed{T.M = T.F \times Distance}$

ASSUMPTIONS OF TORSION :-

i) The shaft should be uniform throughout it's length.
ii) The twist along the shaft. should be uniform
iii) All the diameters before and after the twist should remain straight
iv) The cross-sectional area of the shaft should remain plane, before and after the twist of the shaft.

TORQUE PRODUCE IN A SHAFT :-

When the shaft are subjected to torque then in that case it is subjected to a shear stress which is proportional to the radius of the shaft and the ratio of shear stress (τ) to the radius of the shaft (R) is always constant.



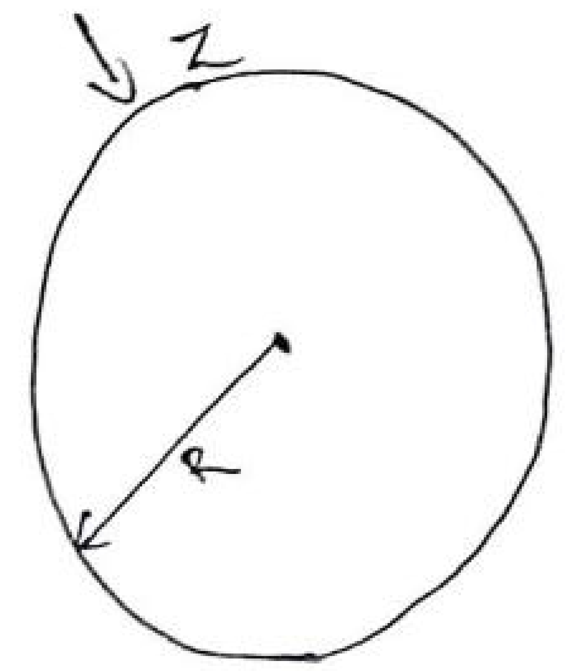
$$\frac{Z}{R} = \text{costant}$$

$$\boxed{\frac{Z}{R} = \frac{Z'}{R'}}$$

EXPRESSION FOR TORQUE IN A SOLID SHAFT

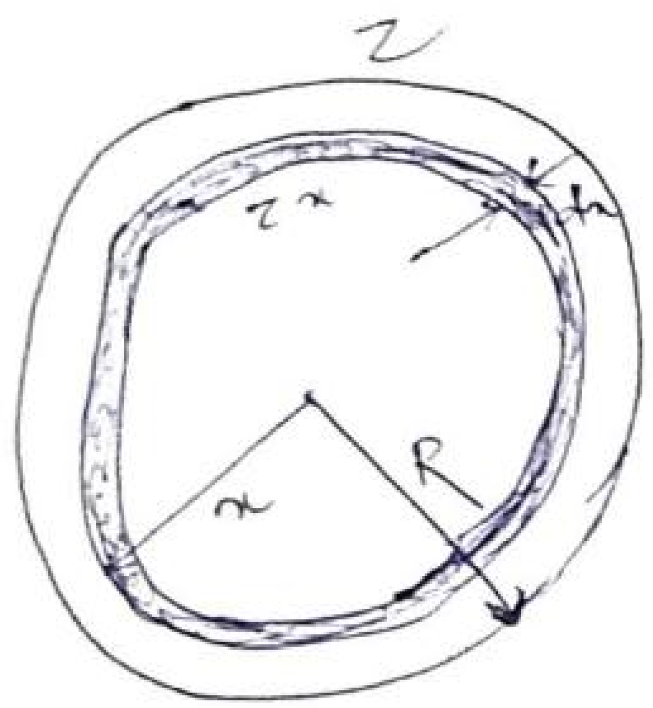
Let us consider a solid shaft having

R = Radius of the shaft
Z = shear stress subjected on the shaft.



If we consider a small element inside the shaft than it will be having

dx = thickness of element
da = area of element



x = radius of element

Z_x = shear stress on the element

da = area of element

dx = thickness of element

Area of the element (da) = Perimeter $\times$ thickness

$$da = 2\pi r \times t$$

$$\boxed{da = 2\pi x \times dx}$$

x = radius
dx = thickness

If we consider the main shaft and the element of the shaft then

$$\boxed{\frac{Z}{R} = \frac{Z_x}{x}}$$

$\therefore$ $Z_x = \frac{Z}{R} \times x$

In this case shear stress = $\frac{\text{Turning force}}{\text{Area}}$

$\Rightarrow$ Turning force = shear stress $\times$ Area.

$\Rightarrow$ $dF = Z_x \times da$

$\Rightarrow$ $dF = \frac{Z}{R} \times x \times 2\pi x \times dx$

$\Rightarrow$ $\boxed{dF = \frac{2\pi Z}{R} \times x^2 \times dx}$

Turning moment = Turning force $\times$ distance

$$dT = \frac{2\pi Z}{R} \times x^2 \times dx \times x$$

$$dT = \frac{2\pi Z}{R} \times x^3 \times dx$$

Integrating the turning moment of the element we get

IMP

$$\int_0^T dT = \int_0^R \frac{2\pi Z}{R} \times x^3 \times dx$$

$\Rightarrow$ $\left[T\right]_0^T = \frac{2\pi Z}{R} \int_0^R x^3 \, dx$ $\left[\int x^n = \frac{x^{n+1}}{n+1}\right]$

$\Rightarrow$ $\left[T - 0\right] = \frac{2\pi Z}{R} \times \left[\frac{x^4}{4}\right]^R$

$\Rightarrow$ $T = \frac{2\pi Z}{R} \times \left[\frac{R^4}{4} - \frac{0^4}{4}\right]$

$\Rightarrow$ $T = \frac{2\pi Z}{\cancel{R}} \times \frac{R^4}{4}$

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$$\Rightarrow T = \frac{2\pi \tau R^3}{4/2}$$

$$\Rightarrow T = \frac{\pi \tau R^3}{2}$$

We know that

$$R = \frac{D}{2}$$

$$\Rightarrow T = \frac{\pi \tau R^3}{2}$$

$$\Rightarrow T = \frac{\pi \tau \left(\frac{D}{2}\right)^3}{2}$$

$$\Rightarrow T = \frac{\pi \times \tau \times D^3}{2 \times 8}$$

$$\Rightarrow \boxed{T = \frac{\pi}{16} \times \tau \times D^3} = \boxed{T = \frac{\pi \tau D^3}{16}}$$

Expression of torque ~~of~~ for solid shaft.

EXPRESSION OF TORQUE FOR A HOLLOW SHAFT

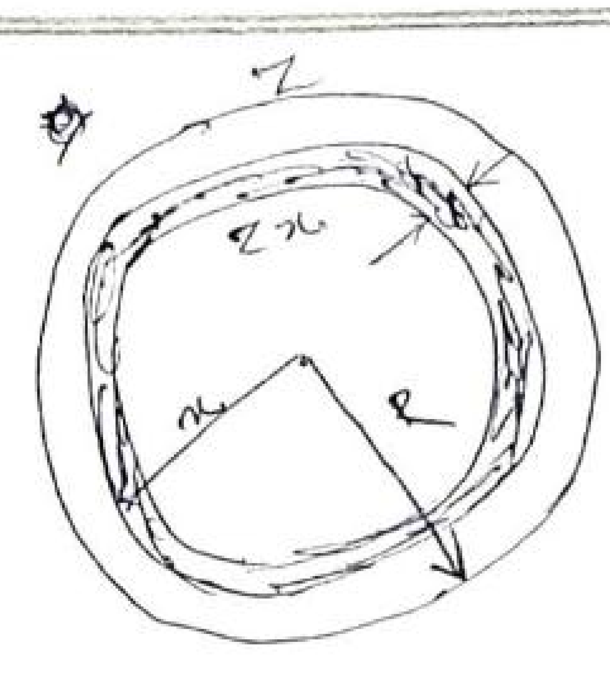
Let us consider a hollow shaft having

R = Radius of Larger circle.

r = ~~small~~ radius of smaller circle.

τ = Shear stress subjected on the shaft

τ_x = Shear stress subjected on the Larger circle smaller circle.



x = radius of element

Area of the element (da)

= Perimeter × thickness

$da = 2\pi x \times t$

$= 2\pi x \times dx$

~~Area~~

If we consider the main shaft and the element of the shaft then

$$\boxed{\frac{\tau}{R} = \frac{\tau_x}{x}} \quad \therefore \tau_x = \frac{\tau}{R} \times x$$

In this case shear stress = $\frac{\text{Turning force}}{\text{Area}}$.

$\Rightarrow$ Turning force = shear stress × Area.

$\Rightarrow \quad dF = \tau_x \times da$

$\Rightarrow \quad dF = \frac{\tau}{R} \times x \times 2\pi x \times dx$

$$\Rightarrow \boxed{dF = \frac{2\pi \tau}{R} \times x^2 \times dx}$$

Turning moment = Turning force × distance.

$$dT = \frac{2\pi \tau}{R} \times x^2 \times dx \times x$$

$$dT = \frac{2\pi \tau}{R} \times x^3 \times dx$$

~~Integrate~~

Integrating turning moment of the element we get

$$\int_0^T dT = \int_r^R \frac{2\pi z}{R} \times x^3 \times dx$$

$$\Rightarrow \left[T\right]_0^T = \frac{2\pi z}{R}\int_r^R x^3\,dx$$

$$\Rightarrow \left[T-0\right] = \frac{2\pi z}{R}\left[\frac{x^4}{4}\right]_r^R$$

$$\Rightarrow T = \frac{2\pi z}{R}\left[\frac{R^4}{4} - \frac{r^4}{4}\right]$$

As $R = \frac{D}{2}$, $r = \frac{d}{2}$

$$\Rightarrow T = \frac{2\pi z}{\frac{D}{2}} \times \left[\frac{\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2}{4}\right]$$

$$\Rightarrow T = \frac{4\pi z}{D} \times \frac{1}{4}\left[\frac{D^4}{16} - \frac{d^4}{16}\right]$$

$$\Rightarrow T = \frac{\pi z}{D}\left[\frac{D^4 - d^4}{16}\right]$$

$$\Rightarrow \boxed{T = \frac{\pi}{16} \times z \times \left(\frac{D^4 - d^4}{D}\right)}$$

Torque for hollow shaft

POWER TRANSMITTED BY THE SHAFT :-

$$P = \frac{2\pi N T}{60}$$

where

P = Power transmitted in KW

N = Speed in RPM

T = Torque in N·m

Q. A solid circular shaft having 50 mm diameter is subjected to a shear stress of 40 MPa. Calculate the Torque.

Given data = $D = 50$ mm.

$z = 40$ MPa $= 40$ N/mm^2

$T = ?$

$$T = \frac{\pi}{16} \times z \times D^3$$

$$= \frac{\pi}{16} \times 40 \times (50)^3 = 981747.70 \text{ N·mm}$$

Q. A solid shaft transmit a torque of 10 KNm If the shear stress is given 45 MPa then calculate the dia of shaft.

$T = 10$ KN·m

$= 10 \times 10^3 \times 10^3$ N·mm $= 10^7$ N·mm

$z = 45$ MPa $= 45$ N/mm.

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$\Rightarrow$ $10^7 = \frac{\pi}{16} \times 45 \times D^3$

$\Rightarrow$ $D^3 = \frac{10^7 \times 16}{\pi \times 45}$

$\Rightarrow$ $D = \sqrt[3]{\frac{10^7 \times 16}{\pi \times 45}}$

$\Rightarrow$ ~~$D = \sqrt[2]{1131768.48}$~~

$\Rightarrow$ $D = 104.21$ mm

Q. A hollow shaft having external dia 80 mm and internal dia 50 mm. is subjected to a shear stress of 45 MPa. Calculate the torque transmitted.

$D = 80$ mm.
$d = 50$ mm
$\tau = 45$ N/mm²

$$T = \frac{\pi}{16} \times \tau \left(\frac{D^4 - d^4}{D}\right)$$

$$= \frac{\pi}{16} \times 45 \times \left(\frac{80^4 - 50^4}{80}\right)$$

$$= 38\ 33602.06 \text{ N.mm}$$

Q. A solid circular shaft of 60 mm diameter is running at 150 RPM. If the shear stress is given 50 MPa, calculate the power transmitted by the shaft.

Data given $D = 60$ mm.
$N = 150$ RPM
$\tau = 50$ MPa $= 50$ N/mm²
$T = ?$
$P = ?$

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 50 \times (60)^3$$

$$= 2120575.04 \text{ N/mm}$$

$$P = \frac{2\pi N T}{60}$$

$$= \frac{2 \times \pi \times 150 \times 2120575.04}{1000 \times 60}$$

$$= 33309.91 \text{ W}$$

$$= 33.309 \text{ KW}$$

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Q. A hollow shaft of external dia 100 mm and internal dia 40 mm. is running at 120 RPM. If the shear stress is given 50 MPa, calculate the power transmitted by the shaft.

$D = 100\,mm$. $\quad \tau = 50\,N/mm^2$

$d = 40\,mm$.

$N = 120\,RPM$

$T = ?$

$P = ?$

$$T = \frac{\pi}{16} \times \tau \left(\frac{D^4 - d^4}{D}\right)$$

$$= \frac{\pi}{16} \times 50 \times \left(\frac{100^4 - 40^4}{100}\right)$$

$$= \frac{9566149.63\ N.mm}{1000}$$

$$= 9566.14\ N.m.$$

$$P = \frac{2\pi N T}{60}$$

$$= \frac{2\pi \times 120 \times 9566.14}{60}$$

$$= 120211.66\ W$$

$$= 120.21\ KW$$

Q. A solid circular shaft of 100 mm diameter is transmitting 120 kW at 150 RPM. Calculate the intensity of shear stress.

$D = 100\,mm$.

$N = 150\,RPM$.

$P = 150 \times 10^3\,W$

$$P = \frac{2\pi N T}{60} \qquad \begin{array}{l} P \text{ in kW} \\ T \text{ in N.m} \end{array}$$

$$\Rightarrow 120 \times 10^3 = \frac{2\pi \times 150 \times T}{60}$$

$$\Rightarrow T = \frac{120 \times 10^3 \times 60}{2\pi \times 150}$$

$$= 7639.43\ N.m.$$

$$= 7.639\ N.m$$

$$= 7.639 \times 10^3\ N.mm.$$

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$\Rightarrow 7.639 \times 10^3 = \frac{\pi}{16} \times \tau \times (100)^3$$

$$\Rightarrow \cancel{\tau = \frac{7.639 \times 16}{100^3}}$$

$$\Rightarrow \tau = \frac{7.639 \times 10^3 \times 16}{100^3 \times \pi}$$

$$= 0.0389\ N/mm^2$$

$$= 0.0389 \times 10^3 = 38.9\ N/mm^2$$

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Q. A solid circular shaft of 120 mm diameter is transmitting 100 KW at 120 RPM. Calculate the shear stress subjected on the shaft.

Data given $D = 120$ mm.
$N = 120$ RPM
$P = 100$ KW

$$P = \frac{2\pi NT}{60} \Rightarrow 100 = \frac{2\pi \times 120 \times T}{60}$$

$$\Rightarrow T = \frac{100 \times 66}{120 \times 2\pi}$$

$$\Rightarrow T = 7.95 \text{ N.m}$$

$$\Rightarrow T = 7.95 \times 10^3 \text{ N.mm}$$

$$T = \frac{\pi}{16} \times Z \times D^3$$

$$\cancel{\frac{\pi}{16}}$$

$$\Rightarrow 7.95 \times 10^3 = \frac{\pi}{16} \times Z \times (120)^3$$

$$\Rightarrow Z = \frac{7.95 \times 10^3 \times 16}{(120)^3 \times \pi}$$

$$= 0.023 \; \cancel{\text{MPa}} \; \text{N/mm}^2$$

Q. A hollow shaft is used to transmit 200 KW at 80 RPM. If the shear stress is 60 MPa and the internal dia is 0.6 of the external diameter then calculate the dia meters of the shaft.

Data given $P = 200$ KW
$N = 80$ RPM
$Z = 60$ MPa
~~D = 0.6 d~~
$d = 0.6D$

$$P = \frac{2\pi NT}{60}$$

$$\Rightarrow 200 = \frac{2\pi \times 80 \times T}{60} \Rightarrow T = \frac{200 \times 60}{2\pi \times 80}$$

$$\Rightarrow T = 23.87 \times 10^3 \text{ N.m}$$

$$T = \frac{\pi}{16} \times Z \times \left[\frac{D^4 - d^4}{D}\right]$$

$$\Rightarrow \cancel{23.87 \times 10^3} = \frac{\pi}{16} \times$$

$$T = \frac{\pi}{16} \times Z \times \left[\frac{D^4 - d^4}{D}\right]$$

$$\Rightarrow T = \frac{\pi}{16} \times 60 \times \left[\frac{D^4 - (0.6D)^4}{D}\right]$$

$$\Rightarrow T = \frac{\pi}{16} \times 60 \times \left[\frac{D^4 - 0.1296D^4}{D}\right]$$

$\Rightarrow T = \frac{\pi}{16} \times 60 \times \frac{\cancel{D}^4(1 - 0.1296)}{\cancel{D}}$

$\Rightarrow T = \frac{\pi}{16} \times 60 \times D^3 \times 0.8704$

$\Rightarrow T = 10.25 D^3$ N.mm

$\Rightarrow T = 10.25 \times 10^{-3}$ N.m.

$P = \frac{2\pi NT}{60}$ $\cancel{\Rightarrow 200 = 2\pi \times 80 \times 10 \times}$

$\Rightarrow 200 = \frac{2\pi \times 80 \times 10.25 \times 10^{-3} D^3}{60}$

$\Rightarrow D^3 = \frac{200 \times 60}{2\pi \times 80 \times 10.25 \times 10^{-3}}$

$\Rightarrow D = \sqrt[3]{2329.09}$

$\Rightarrow D = 13.25$ mm

$d = 0.6 D$

$= 0.6 \times 13.25 = 7.95$ mm

Q. A solid steel shaft transmit 100 kW at 160 RPM. If the shear stress is given 70 MPa. calculate the dia of the shaft. consider the ~~Data given~~ max torque exceeds the mean torque by 20%.

Data given $P = 100$ kW
$N = 160$ RPM
$\tau = 70$ MPa

$P = \frac{2\pi NT}{60} \Rightarrow 100 = \frac{2\pi \times 160 \times T}{60}$

$\Rightarrow T = \frac{100 \times 60}{2\pi \times 160}$

$\Rightarrow T = 5.96$ kN.m

$\cancel{\Rightarrow T = 5.96 \times 10^3 \text{ N.m}}$

$\Rightarrow T = 5.96 \times 10^3 \times 10^3$ N.mm

$\Rightarrow T = 5.96 \times 10^6$ N.mm

$\cancel{T = \frac{\pi}{16} \times \tau \times D^3}$

$\cancel{\Rightarrow 5.96 \times 10^6 =}$

$T_{max} = (100\% + 20\%) T_{mean}$

$\Rightarrow T_{max} = 120\% \cdot T_{mean}$

$\Rightarrow T_{max} = \frac{120}{100} T_{mean}$

$\Rightarrow T_{max} = 1.2 T_{mean}$

$\Rightarrow T_{max} = 1.2 \times 5.96 \times 10^6$

$= 7152 \times 10^3$ N.mm

$T = \frac{\pi}{16} \times \tau \times D^3$

$\Rightarrow D^3 = \frac{7152 \times 10^3 \times 16}{70 \times \pi}$

$\Rightarrow D = \sqrt[3]{520354.81} = 80.43.$

Q A solid circular shaft transmit 110 KW at 90 RPM. If the shear stress is given 65 MPa then calculate the dia of the shaft. consider the max torque exceeds the mean torque by 45%.

Data given $P = 1100 \times 10^3$ W

$N = 90$ RPM

$Z = 65$ N/mm²

$P = \frac{2\pi N T}{60} \Rightarrow T = \frac{60 \times 110 \times 10^3}{2\pi \times 90}$

$\Rightarrow T = 11671.36$ N.m.

$\Rightarrow T = 11671.36 \times 10^3$ N.mm

$T_{max} = 145\% . T_{mean}$

$= \frac{145}{100} T_{mean}$

$= 1.45 \times 11671.36 \times 10^3$

$= 16923472$ N.mm.

$T = \frac{\pi}{16} \times Z \times D^3$

$\Rightarrow D = \sqrt[3]{\frac{16923472 \times 16}{\pi \times 65}}$

$\Rightarrow D = 109.66$ mm,

REPLACING A SHAFT :-

Sometimes we are required to replace a solid shaft with a hollow shaft or a hollow shaft with a solid shaft. In both the cases we have to make sure that the torque transmitted and the shear stress are constant. and equal

Q A solid steel shaft of 60 mm dia is to be replaced by a hollow shaft having the internal dia of 85mm. Calculate the external dia of hollow shaft.

Torque transmitted Solid shaft = Torque transmitted Hollow shaft.

$T = T$

$\frac{\pi}{16} \times Z \times D^3 = \frac{\pi}{16} \times Z \times \left[\frac{D^4 - d^4}{D}\right]$

$\Rightarrow (60)^3 = \left[\frac{D^4 - (85)^4}{D}\right]$

$\Rightarrow (60)^3 D = D^4 - (85)^4$

$\Rightarrow 216000 D = D^4 - 52200625$

$\cancel{\Rightarrow 216000 D + 52200625 = D^4}$

$\Rightarrow 52200625 = D^4 - 21600 D$

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Q. A solid steel shaft having 80 mm dia is to be replaced by a hollow shaft of external dia 100 mm & Determine the internal dia of the hollow shaft.

Torque transmitted by solid shaft (T) = Torque transmitted by hollow shaft (T)

$$\frac{\pi}{16} \times \cancel{\tau} \times D^3 = \frac{\pi}{16} \times \cancel{\tau} \times \left[\frac{D^4 - d^4}{D}\right]$$

$$\Rightarrow (80)^3 = \frac{(100)^4 - d^4}{100}$$

$$\Rightarrow 80^3 \times 100 = 100^4 - d^4$$

$$\Rightarrow d^4 = 100^4 - 80^3 \times 100$$

$$\Rightarrow d = \sqrt[4]{100^4 - 80^3 \times 100}$$

$$\Rightarrow d = 83.58$$

Q. A solid steel shaft of 60 mm dia is to be replaced by a hollow shaft. The internal dia of hollow shaft is equal to half of the external dia. of the hollow shaft. Determine the external and internal dia of hollow shaft.

Solid = 60 mm.

$$d_h = \frac{1}{2} D_h$$

$$\Rightarrow d_h = 0.5 D_h$$

Torque transmitted by solid shaft (T) = Torque transmitted hollow shaft (T)

$$D^3 = \frac{D^4 - d^4}{D}$$

$$\Rightarrow (60)^3 = \frac{D^4 - 0.5 D^4}{D}$$

$$\Rightarrow \cancel{60^3 \times D = D^4 - 0.5 D^4}$$

$$\Rightarrow \cancel{60^3 \times D = 0.5 D^4}$$

$$\Rightarrow \cancel{60^3 = \frac{0.5 D^4}{D}}$$

$$\Rightarrow \cancel{D = \sqrt[3]{60^3}} = 60 \text{ mm}$$

$$d = \cancel{0.5 \times 60 = 30 \text{ mm}}$$

$$\Rightarrow 60^3 \times D = D^4 - 0.0625 D^4$$

$$\Rightarrow 60^3 \times D = D^4 (1 - 0.0625)$$

$$\Rightarrow 60^3 = \frac{D^3 \times 0.9375}{}$$

$$\Rightarrow D = \sqrt[3]{\frac{60^3}{0.9375}} = 61.30 \text{ mm}$$

$$d = \frac{1}{2} \times 61.30 = 30.65$$

(M-5) (THEORY OF SIMPLE BENDING)

When a beam is subjected to a bending moment then the process of bending starts. The cross-section of the beam offers a resistance to the process of bending. If the resistance is greater than the bending moment than the process of bending stops. This resistance is known as bending stress.

THEORY OF SIMPLE BENDING :- IMP

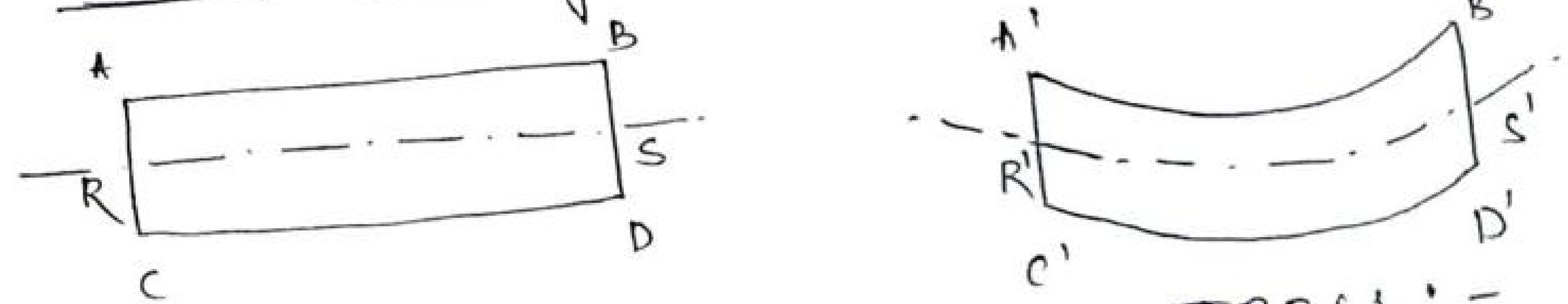
Let us consider a small length of a beam ~~two~~ subjected to bending moment. Now consider section AB and CD in the section of the beam which are parallel to the axis RS.

Let the beam is subjected to a bending moment and the length of the section before bending should not remain same after bending. The top layer of the beam is subjected to compression ~~tension~~ and it's length decreses ~~increases~~ from AB to A'B'. The bottom layer of the beam is subjected to ~~compression~~ tension and it's length increses from CD to C'D'. There will be one layer present in the centre of the beam which will remain ~~unaffected~~ unaffected by the tension or compression. The length of this

layer will not change at all. This layer is known as neutral axis or neutral layer.

A little consideration will show that as we move from the top layer to the central layer than the length will goes on increasing. And if we move from the bottom layer to the central layer than the length will keep decreasing.

Before Bending | After Bending



CALCULATION OF BENDING STRESS :-

$$\boxed{\sigma_b = \frac{E}{R} \times y}$$

Where σ_b = bending stress

E = Young's Modulus

R = Radius of the object.

y = Half of the diameter of the material to cover the curvature

Q. A steel wire of 5 mm diameter is bent into a circular shape 5 m radius. determine the bending stress in the wire. Take $E = 200\ GPa$

Q. A copper wire of 2 mm dia is wrapp around on a drump. If the bending stress is given 80 MPa, Calculate the radius of the drum. Take $E = 100\ GPa$

Q. A metallic rod of 10 mm diameter is bent into a circular form of radius 6m, If the bending stress is given 125 MPa, Calculate the Young's modulus of the material.

Q.1 Data given $D = 5$, $y = \frac{5}{2} = 2.5\ mm$

$R = 5m = 5000\ mm$

$E = 200\ GPa = 200\times10^3\ N/mm^2$

$$\sigma_b = \frac{E}{R}\times y$$

$$= \frac{200\times10^3}{5000}\times 2.5 = 100\ N/mm^2$$

Q.2 Data given $d = 2\ mm$, $1\ mm = y$

$\sigma_b = 80\ MPa = 80\ N/mm^2$

$E = 100\ GPa = 100\times10^3$

$$\sigma_b = \frac{E}{R}\times y$$

$$\Rightarrow 80 = \frac{100\times10^3}{R}\times 1$$

$$\Rightarrow R = \frac{100\times10^3}{80}$$

$$\Rightarrow R = 1250\ mm.$$

Q.3 Data given $D = 10\ mm$, $y = \frac{10}{2} = 5\ mm$

$R = 6m = 6000\ mm$

$\sigma_b = 125\ MPa = 125\ N/mm^2$

$$\sigma_b = \frac{E}{R}\times y$$

$$\Rightarrow 125 = \frac{E}{\cancel{6000}\ 1200}\times \cancel{5}$$

$$\Rightarrow E = 1200\times125 = 15\times10^4\ N/mm^2$$

SECTION MODULUS:- (Z)

$$Z = \frac{I}{y} = \frac{\text{Moment of inertia}}{\text{half of the diameter}}$$

STRENGTH SECTION (M):-

$$\boxed{M = \sigma_b\times Z}$$

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(CH-6) (PRINCIPLE STRESS & PRINCIPLE PLANE)

PRINCIPLE PLANE :-

The plane in which the magnitude of shear stress is equal to "zero", that plane is known as principle plane.

In other words in priciple planes the shear stresses is always 'zero'

The stresses which are subjected on the priciple plane is known as principle stresses.

* There are various cases where stresses are subjected to an inclined plane or oblique plane

Case-1 :- (σx)

An oblique section of a body subjected to a direct stress. (σx)

Case-2 :- (σx, σy)

An oblique section of a body subjected to a two direct stress.

Case 3 :- (σx, τx)

An oblique section of a body subjected to a direct stress and a shear stress.

Case 4 :- (σx, σy, τxy)

An oblique section of a body subjected to a two direct stress and a shear stress

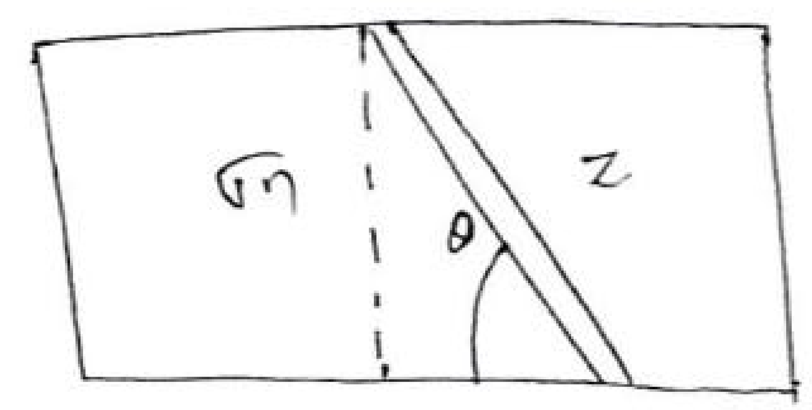
For calculating the stresses on an oblique plane there are two methods

i) Analytical method.

ii) Graphical Method.

i) Analytical method for calculation of stresses in an Inclined or oblique plane.

Case-1 :- An oblique section of a body subjected to a direct stress (σx)



In this case the inclined section will be acted upon by two stress. They are

a) Normal stress (σn)

b) Shear stress (τ)

Normal stress (σn)

i) $\sigma n = \frac{\sigma}{2} - \frac{\sigma}{2} \cos 2\theta$

Shear stress (τ)

ii) $\tau = \frac{\sigma}{2} \sin 2\theta$

Resultant stress (σR)

iii) $\sigma R = \sqrt{(\sigma n)^2 + (\tau)^2}$

Q. A wooden bar is subjected to a tensile stress of 5 MPa. Calculate the value of normal stress shear stress if it's makes angle of 25° with the plane.

Data given $\sigma_t = 5\text{ MPa} = 5\text{ N/mm}^2$

$\theta = 25°C$

$\sigma_n = \frac{5}{2} - \frac{5}{2}\cos 2\times25°$

$= 2.5 - \frac{5}{2}\times 0.64 = 0.9\text{ N/mm}^2/\text{MPa}$

$\tau = \frac{5}{2}\sin 2\times25° = 1.91\text{ MPa}$

Q. A wooden bar is subjected to a normal stress of 0.5 MPa and shear stress of 1.25 MPa. If the cross section is 200 × 100 mm then calculate the direct stresses in both the cases. Also calculate the ~~safe~~ value of force, Take $\theta = 60°$

Data given $\sigma_n = 0.5\text{ MPa} = 0.5\text{ N/mm}^2$

$\tau = 1.25\text{ MPa} = 1.25\text{ N/mm}^2$

$\theta = 60°$

1) $\sigma_n = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta$

$\Rightarrow 0.5 = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 120$

$\Rightarrow 0.5 = \frac{\sigma}{2} + \frac{\sigma}{4}$

$\Rightarrow \frac{2\sigma+\sigma}{4} = 0.5 \Rightarrow 3\sigma = 0.5\times4$

$\Rightarrow \sigma = \frac{0.5\times4}{3} = 0.66\text{ MPa}$

$\tau = \frac{\sigma}{2}\sin 2\theta$

$\Rightarrow 1.25 = \frac{\sigma}{2}\sin 120$

$\Rightarrow 1.25 = \frac{\sigma}{2}\times\frac{\sqrt{3}}{2}$

$\Rightarrow \sigma = 1.25\times\frac{4}{\sqrt{3}} = 2.88\text{ MPa}$

$\text{stress} = \frac{\text{Force}}{\text{Area}}$

$\Rightarrow \text{Force} = \text{Stress}\times\text{Area}.$

$= 0.66\times2\times10^4$ (Smallest value of stress) for safe value

$= 13200\text{ N}$

max – Largest value

Q. A wooden bar is subjected to a shear stress of 1.3 MPa. The cross section is 100 mm × 100 mm making an angle 60° with the plane then calculate

i) stress

ii) Force

Data given $\tau = 1.3\text{ N/mm}^2$

$\theta = 60°$

$A = 100\times100 = 10^4\text{ mm}^2$

$\tau = \frac{\sigma}{2}\sin 2\theta$

$\Rightarrow 1.3 = \frac{\sigma}{2}\sin 120°$

$\Rightarrow \sigma = 1.3\times\frac{4}{\sqrt{3}} = 3$

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$$\sigma_n = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta$$

$$= \frac{3}{2} - \frac{3}{2}\cos 2\times 60^\circ$$

$$= 1.5 + \frac{3}{4} = 2.25$$

$\text{Force} = \frac{\text{Stress}}{\text{Area}}$

Force = stress × Area

$= 3\times 10^4$ N

Case-2 :- An oblique section of a body subjected to a two direct stresses.

1) Normal stress $\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$

2) shear stress $\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$

3) Resultant stress $\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$

4) Maximum shear stress

$$\tau_{max} = \pm\frac{\sigma_x - \sigma_y}{2}$$

Q Determine the normal stress, shear stress, resultant stress for an inclined section of a body subjected to two tensile stresses of 200 MPa and 100 MPa making an angle of 30° with the plane.

Data given $\sigma_x = 200$ N/mm²

$\sigma_y = 100$ N/mm²

$\theta = 30^\circ$

$$\sigma_n = \frac{200+100}{2} - \frac{200-100}{2}\cos 2\times 30^\circ$$

$= 125$ MPa

$$\tau = \frac{200-100}{2}\sin 2\times 30$$

$= 50\times\frac{\sqrt{3}}{2} = 43.30$ MPa

$$\sigma_R = \sqrt{(125)^2 + (43.30)^2} = 132.28 \text{ MPa}.$$

Q An inclined section of a body making an angle of 25° with the plane is subjected to a tensile stress of 100 MPa and a compressive stress of 50 MPa. Calculate σ_n, τ, σ_R, τ_{max} also calculate the direction of resultant stress.

Data given $\theta = 25^\circ$

$\sigma_x = 100$ MPa

$\sigma_y = -50$ MPa

$$\sigma_n = \frac{100-50}{2} - \frac{150}{2}\cos 50$$

$= 25 - 75\cos 50 = -23.20$ MPa

$$\tau = \frac{100+50}{2}\sin 50^\circ = 57.45 \text{ MPa}$$

$$\sigma_R = \sqrt{(-23.2)^2 + (57.45)^2} = 61.99 \text{ MPa}$$

$$\tau_{max} = \pm\frac{100+50}{2} = \pm 75 \text{ MPa}$$

Direction of resultant stress

$$\tan\theta = \frac{\tau}{\sigma_n}$$

$\Rightarrow \tan\theta = \frac{57.45}{-23.20}$

$\Rightarrow \tan\theta = -2.47$

$\Rightarrow \theta = \tan^{-1}(-2.47)$

$\Rightarrow \theta = -67^{0}.95^{0}$

Q An inclined body making an angle of 55^0 is subjected to two tensile stresses of 150 MPa and 50 MPa. Calculate σ_n, τ, σ_R, τ_{max}

Data given $\theta = 55^0$

$\sigma_x = 150\ N/mm^2$

$\sigma_y = 50\ N/mm^2$

$$\sigma_n = \frac{200}{2} - \frac{100}{2}\cos 110^0$$

$= 100 - 50\cos 110^0 = 117.10$ MPa.

$\tau = \frac{100}{2}\sin 110^{\circ} = 46.98$ MPa

$\sigma_R = \sqrt{(117.10)^2 + (46.98)^2} = 126.17$ MPa

$\tau_{max} = \pm\frac{150-50}{2} = \pm 50$ MPa

Case 3:- An oblique section of a body is subjected to a direct stress and a shear stress.

1) Normal stress

$$\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

2) Shear stress

$$\tau = \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

3) Resultant stress σ_R

$$\sigma_R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2}$$

4) Maximum shear stress. (τ_{max})

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2}$$

5) Maximum Principal stress.

$$\sigma_{max} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2}$$

6) Minimum Principal stress

$$\sigma_{min} = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2}$$

7) Direction of max shear stress.

$$\tan 2\theta_R = \frac{\sigma_x}{2\tau_{xy}}$$

$\theta_R = ?$

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Q An element in a body is subjected to a tensile stress of 100MPa along with a shear stress of 25MPa ~~along~~ in the clockwise direction making an ~~ago~~ angle of 20°. with the plane. Calculate σ_n, τ and τ_{max}.

Data given $\sigma_x = 100\ N/mm^2$

$\tau_{xy} = 25\ N/mm^2$

$\theta = 20°$

$\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

$= \frac{100}{2} - \frac{100}{2}\cos 40° - 25 \times \sin 40°$

$= -4.37$ MPa or N/mm^2

$\tau = \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$

$= \frac{100}{2}\sin 40° - 25\cos 40°$

$= 12.98$ MPa / N/mm^2

$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2}$

$\cancel{= \sqrt{(4.37)^2 + (12.98)^2}}$

$\cancel{= 12.22}$

$= \sqrt{(50)^2 + (25)^2}$

$= 55.90$ MPa

Q An element is subjected to a tensile a stress of 150MPa along with a shear stress of 50MPa. in the ~~anti~~. anticlockwise direction in ~~the anti~~ making an angle of 40° with the plane. Calculate σ_n, τ, τ_{max} and the direction of max shear stress.

Data given $\sigma_x = 150$ MPa

$\tau_{xy} = -50$ MPa

$\theta = 40°$

$\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

$= \frac{150}{2} - \frac{150}{2}\cos 80° + 50\sin 80°$

$= 111.21$ MPa.

$\tau = \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$

$= 75\sin 80 + 50\cos 80°$

$= 82.54$ MPa.

$\tau_{max} = \sqrt{(75)^2 + (-50)^2}$.

$= 90.13$

$\cancel{\tan 2\theta =}$

$\tan 2\theta = \frac{2\tau_{xy}}{2\sigma_x}$

$\Rightarrow \tan 2\theta = \frac{50}{2 \times 150}$

$\cancel{\tan 2\theta} = \tan 2\theta_R = \frac{\sigma_x}{2\tau_{xy}}$

$\Rightarrow \tan 2\theta_R = \frac{150}{2 \times 50}$

$\Rightarrow 2\theta = \tan^{-1}(1.5)$

$\Rightarrow \theta_R = \frac{56.30}{2}$

$\Rightarrow \theta_R = 28.15^\circ$

Q. An element is subjected to a compressive stress of 200 MPa and a clockwise shear stress of 50 MPa. Making an angle of 35° with the plane calculate σ_n, τ, τ_{max}

Data given $\sigma_x = -200 N/mm^2$

$\tau_{xy} = 50 N/mm^2$

$\theta = 35^\circ$

$\sigma_n = -\frac{200}{2} + \frac{200}{2}\cos 70 - 50 \sin 70$

$= -112.78$ MPa.

$\tau = \frac{\sigma_x}{2} - \frac{200}{2}\sin 70 - 50 \cos 70$

$= -111.07$ MPa

$\tau_{max} = \sqrt{(-100)^2 + (50)^2}$

$= 111.80$ MPa

Case 4:- An oblique section of a body subjected to two direct stress and one shear stress.

i) $\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

ii) $\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$

iii) $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$

iv) Direction of τ_{max}

$\tan 2\theta_R = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$

Q. An element is subjected to 2 tensile stresses of 250 MPa and 100 MPa making an angle of 20° along with a shear stess of 25 MPa in the clockwise direction, calculate σ_n, τ

Data given $\sigma_x = 250 N/mm^2$

$\sigma_y = 100 N/mm^2$

$\theta = 20^\circ$

$\tau_{xy} = 25 N/mm^2$

$\sigma_n = \cancel{250} \frac{350}{2} - \frac{150}{2}\cos 40^\circ - 25 \sin 40$

$= 101.47$ MPa

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$\tau = \frac{150}{2} \sin 40^\circ - 25\cos 40^\circ$

$= 29.05\ MPa$

Maximum principal stress (σ_{max})

$= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Minimum principal stress (σ_{min})

$= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Data given

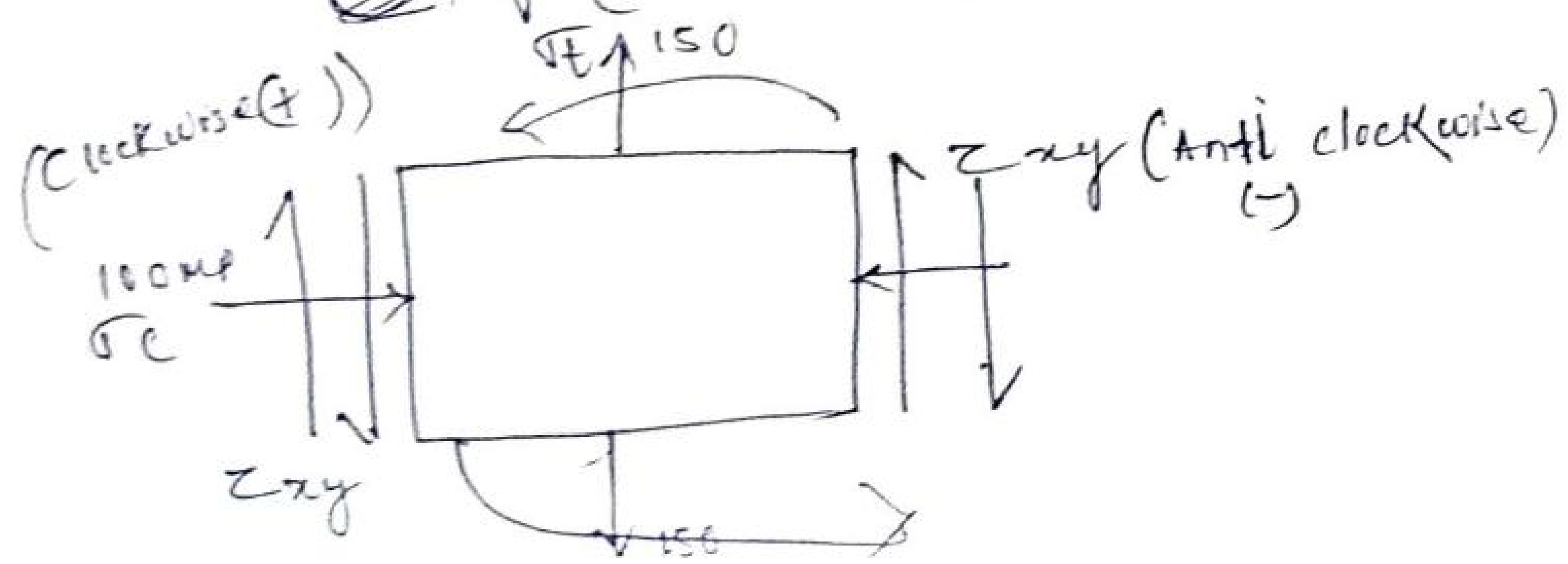
$\sigma_x = 400$

$\sigma_y = 150$

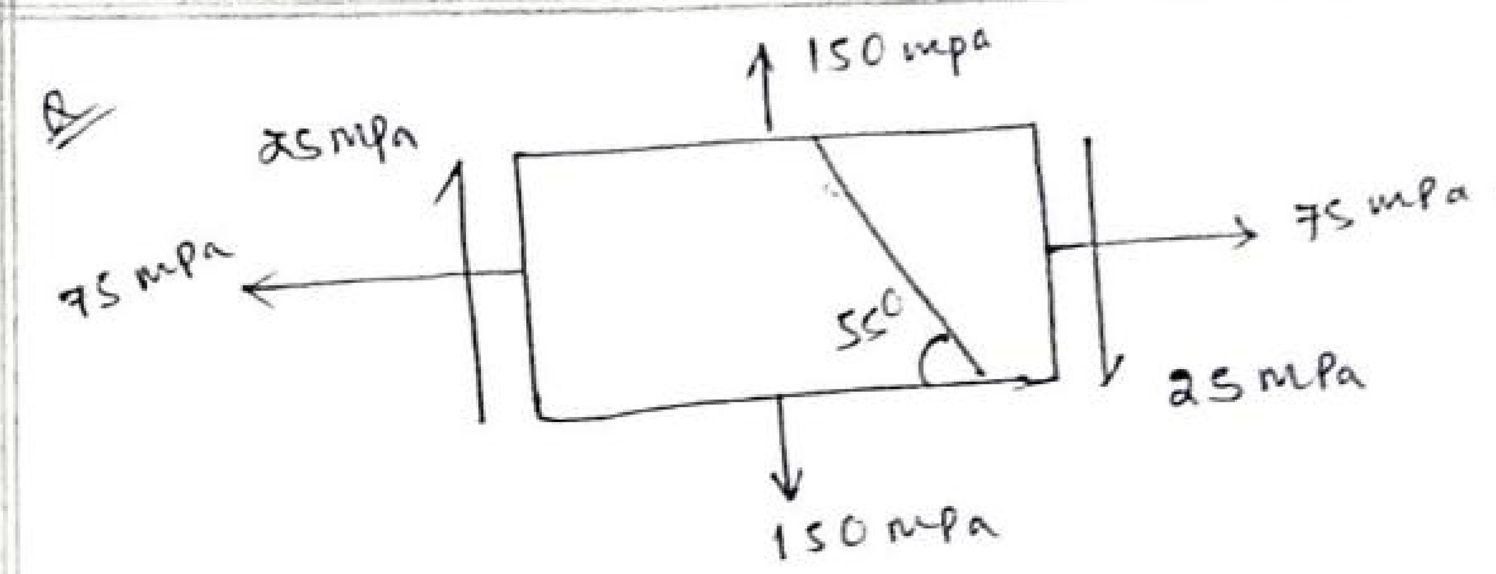
$\tau_{xy} = -100$

$\sigma_{max} = 275 + \sqrt{(125)^2 + (100)^2} = 435.07\ MPa$

$\sigma_{min} = 275 - \sqrt{(125)^2 + (100)^2} = 114.92\ MPa.$



Q



$\theta =$

~~$\sigma_n = \frac{75}{2} - \frac{150}{2}\cos 110^\circ - 25 \times \sin 110^\circ$~~

~~$= 39$~~

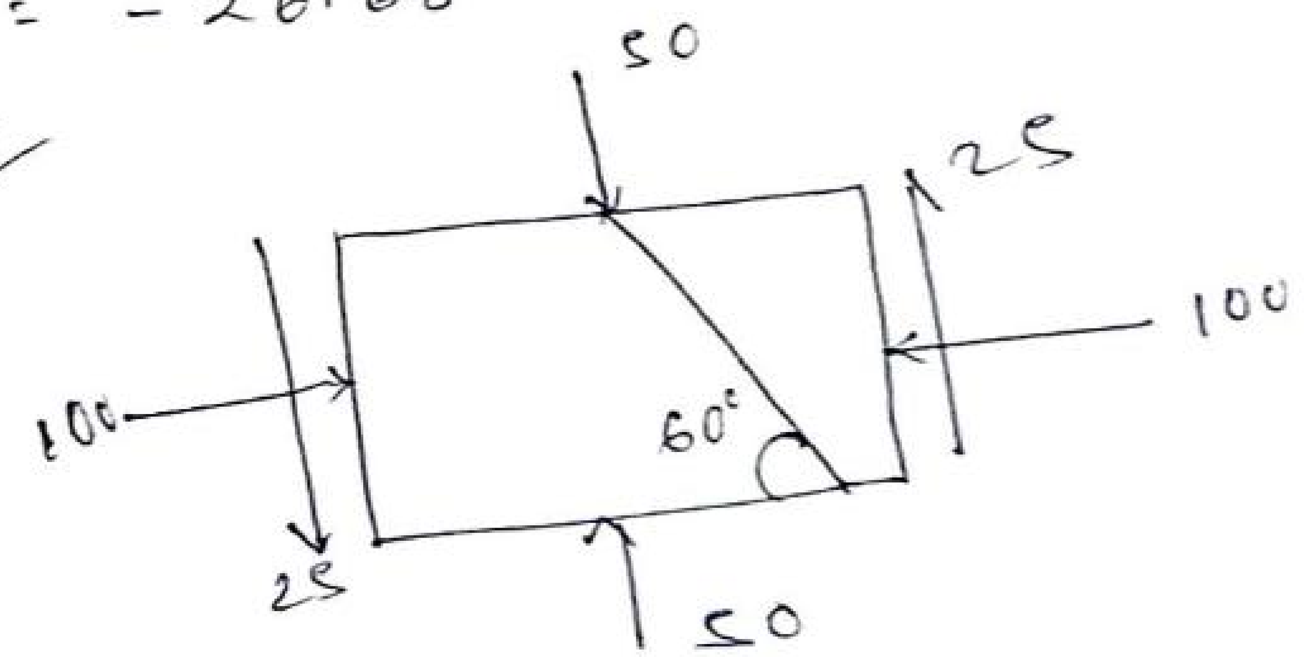
$\sigma_n = \frac{225}{2} - \frac{75-150}{2}\cos 110 - 25\sin 110^\circ$

$= 76.18$

$\tau = \frac{-75}{2}\sin 110^\circ - 25\cos 110^\circ$

$= -26.68$

Q Doubt



$\sigma_n = \frac{150}{2} + \frac{150}{2}\cos 120^\circ + 25\sin 120^\circ$

$= 75 - 75\cos 120^\circ + 25\sin 110^\circ$

$= 134.15$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \frac{-100-50}{2} - \frac{-100+50}{2} \cos 120 + 25 \sin 120$$

$$= 75 + 25 \cos 120^\circ + 25 \sin 120^\circ$$

$$= \cancel{55.39} - 65.84$$

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$= \frac{-100+50}{2} \sin 120 - 25 \cos 120$$

$$= -25 \sin 120^\circ - 25 \cos 120^\circ$$

$$= -34.150$$

GRAPHICAL METHOD FOR CALCULATION OF STRESS

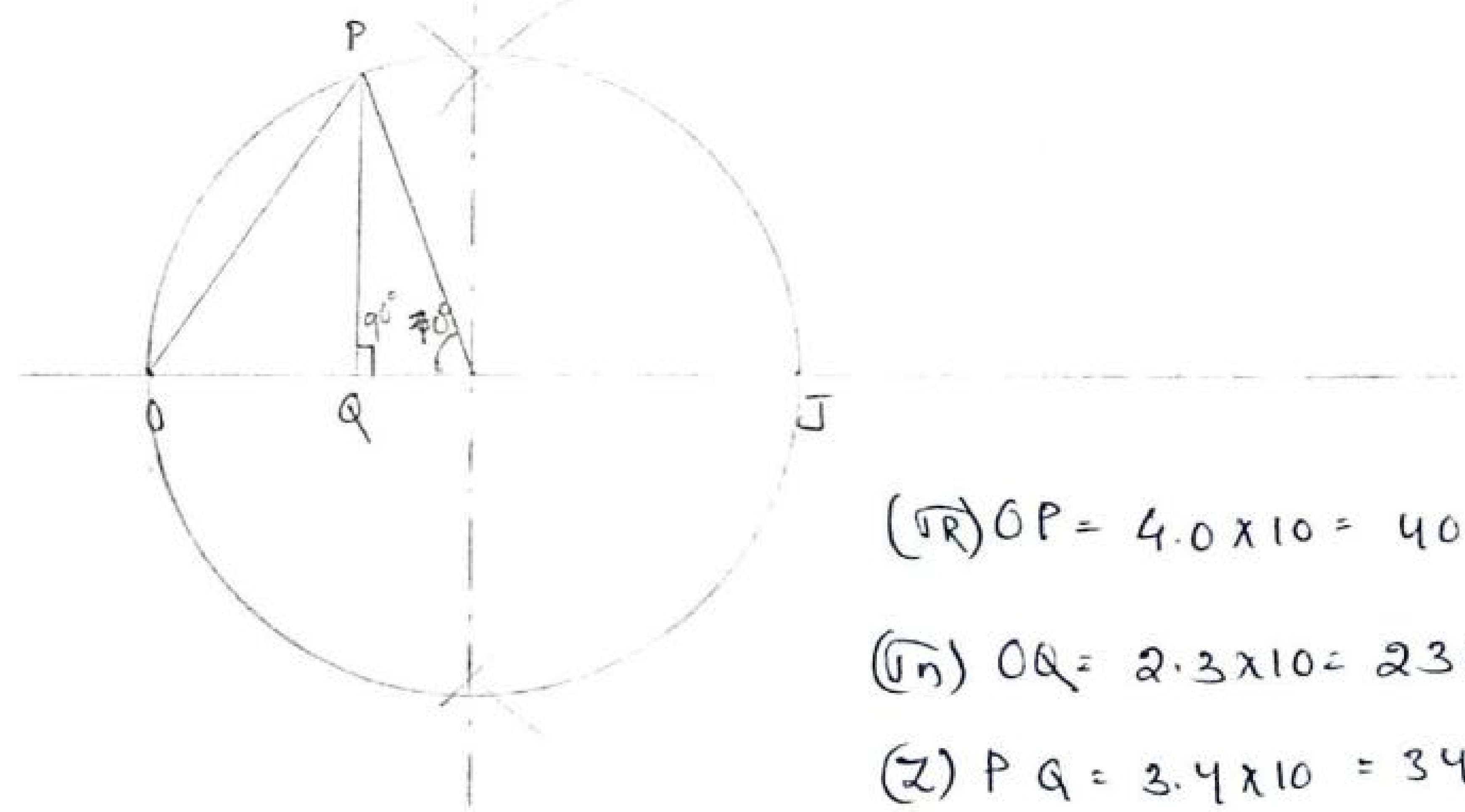
MOHR CIRCLE OF STRESSES:– It is the graphical representation of stresses. In other words it can be defined as the graphical method used for the evaluation of principal stress and principal strain.

Case-1:– When the inclined body is subjected to one direct stress.

1) Take a point 'O' and draw a line through it.
2) Cut 'OJ' equal to the stress towards right on left towards ~~or~~ right. Left of the point.
(Tensile = right, compressive = ~~right~~ left)

3) Bisect 'OJ' at C
4) Taking 'C' as the centre and OC as radius & draw a circle.
5) Extend 2θ angle at 'C' in the clockwise direction.
6) Extend the angle making a point 'P' at the circle.
7) At 'P' draw perpendicular 'PQ' to the line.
8) Join OP
9) OQ will be give you normal stress (σ_n)
QP will " " " shear stress (τ)
OP " " " " Resultant stress (σ_R)

Q. Data given:– $\sigma_n = 70$ MPa $= \frac{70}{10} = 7$ cm
$\theta = 30^\circ \qquad 2\theta = 2 \times 35^\circ = 70^\circ$



(σ_R) OP = 4.0 × 10 = 40 MPa

(σ_n) OQ = 2.3 × 10 = 23 MPa

(τ) PQ = 3.4 × 10 = 34 MPa.

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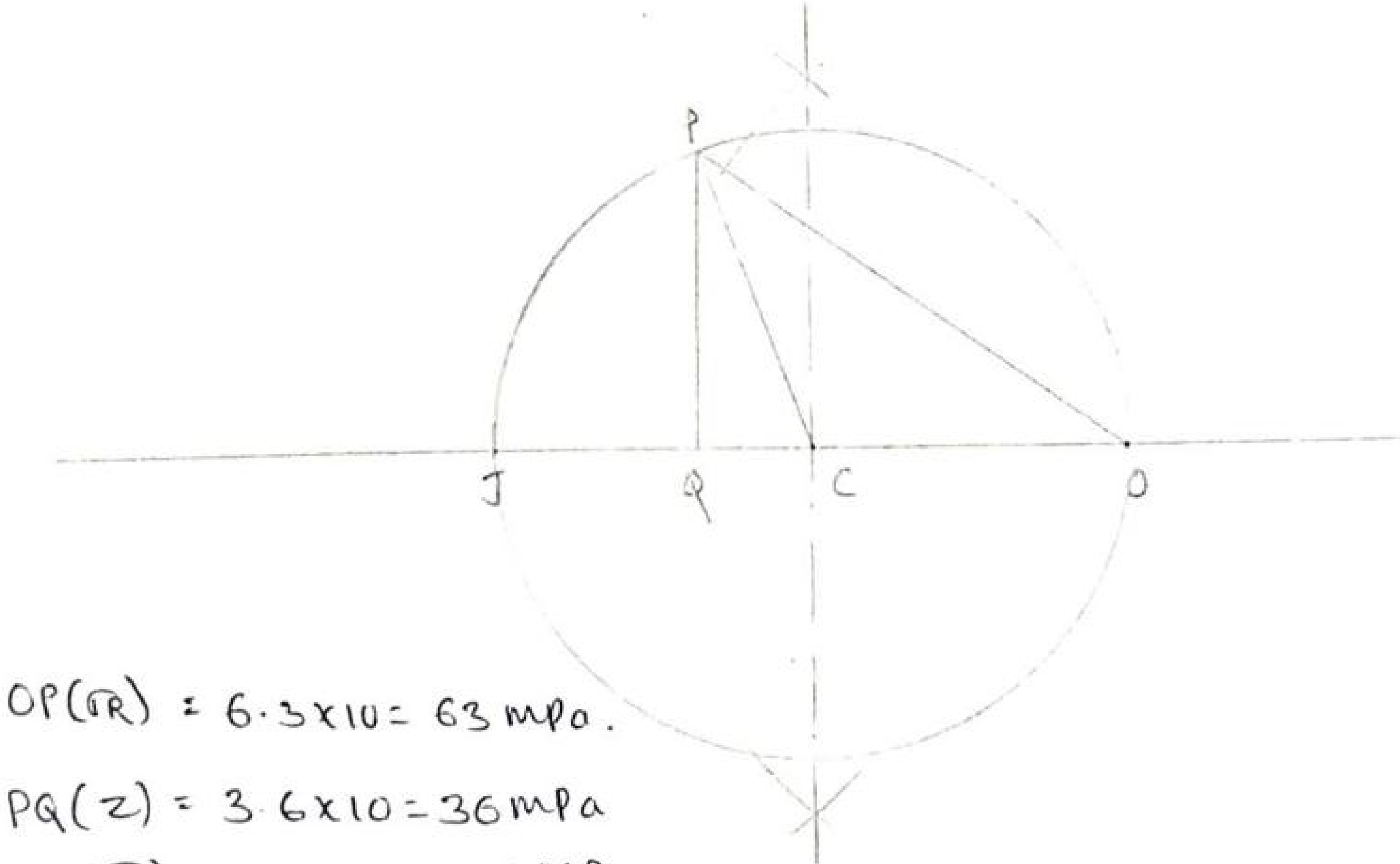
Q Data given :- $\sigma = 75\,MPa = \frac{75}{10} = 7.5$ (compressive)

$\theta = 35^\circ$ $\quad 2\theta = 2 \times 35^\circ = 70^\circ$



$OP(\sigma_R) = 6.3 \times 10 = 63\,MPa$.

$PQ(z) = 3.6 \times 10 = 36\,MPa$

$OQ(\sigma_n) = 5.3 \times 10 = 53\,MPa$

Case-2 :- when an inclined body is subjected to two direct stresses.

1) Take a point 'O' draw a line through it.

2) Cut 'OJ' and 'OK' equal to the stress towards right and left side of 'O' according to its nature.

3) Bisect 'JK' at 'C'

4) Taking 'C' as centre and CJ at radius and draw the mohr circle.

5) At C draw 2θ angle in clockwise direction

6) Mark point 'P' on circle

7) Through P draw perpendicular PQ

8) Join OP.

9) Measure $OQ(\sigma_n)$, $PQ(z)$ and $OP(\sigma_R)$

Q Data given $\sigma_x = 150\,MPa$ tensile $= \frac{150}{10} = 15\,cm$.

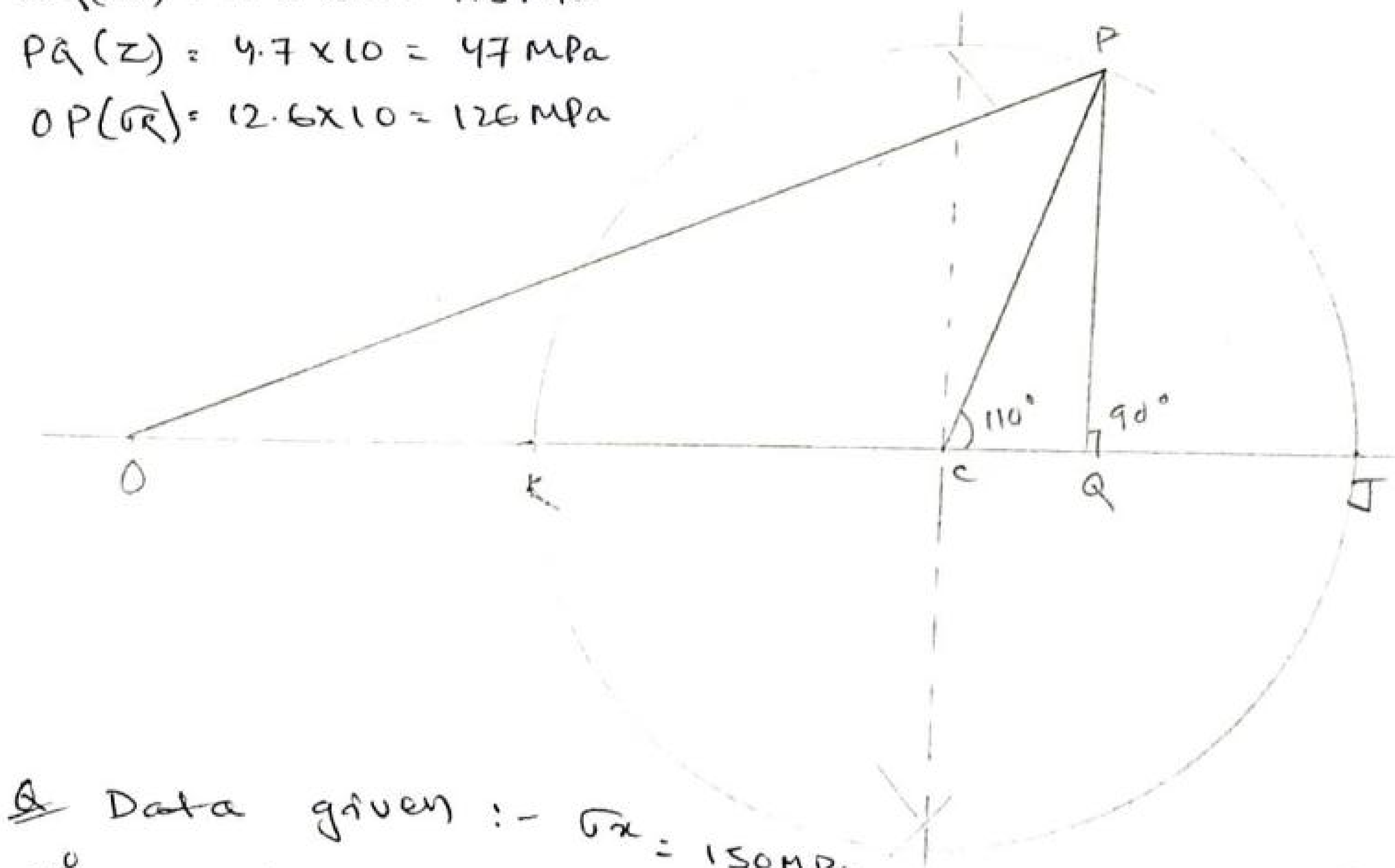
$\sigma_y = 50\,MPa$ tensile $= \frac{50}{10} = 5\,cm$.

$\theta = 55^\circ$ $\quad 2\theta = 2 \times 55^\circ = 110^\circ$

$OQ(\sigma_n) = 11.8 \times 10 = 118\,MPa$

$PQ(z) = 4.7 \times 10 = 47\,MPa$

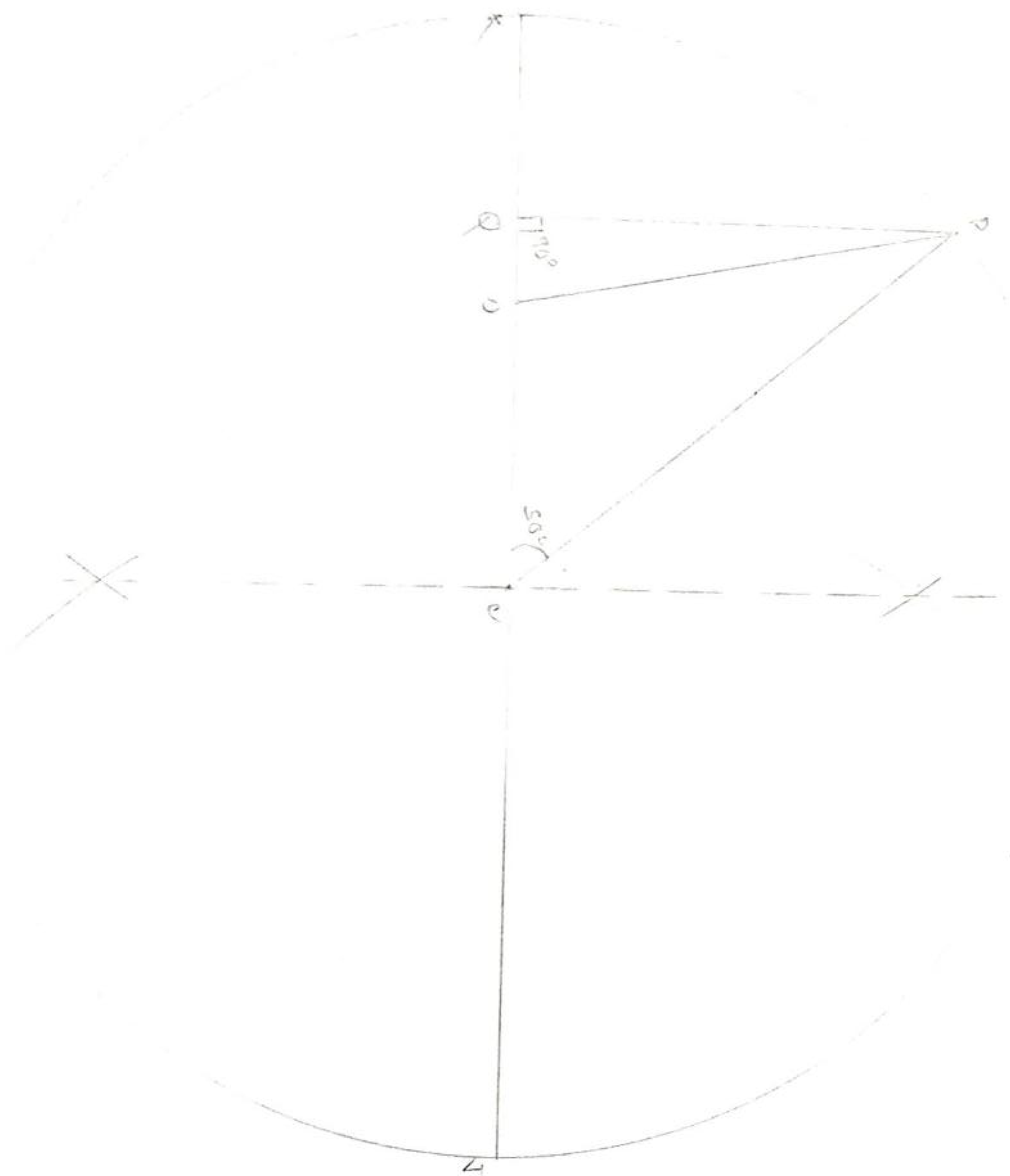
$OP(\sigma_R) = 12.6 \times 10 = 126\,MPa$



Q Data given :- $\sigma_x = 150\,MPa = \frac{150}{10} = 15\,cm$ (tensile)

$\theta = 25^\circ$ $\quad 2\theta = 50^\circ$

$\sigma_y = 50\,MPa$ (compressive) $= \frac{50}{10} = 5\,cm$



$PQ = 5.6 \times 10 = 56$ MPa (σ_n)

$CQ = 2.4 \times 10 = 24$ MPa (τ)

$OP = 6.2 \times 10 = 62$ MPa (σ_R)

Data given $\sigma_x = 100$ MPa (compressive) $= \frac{100}{10} = 10$ cm.

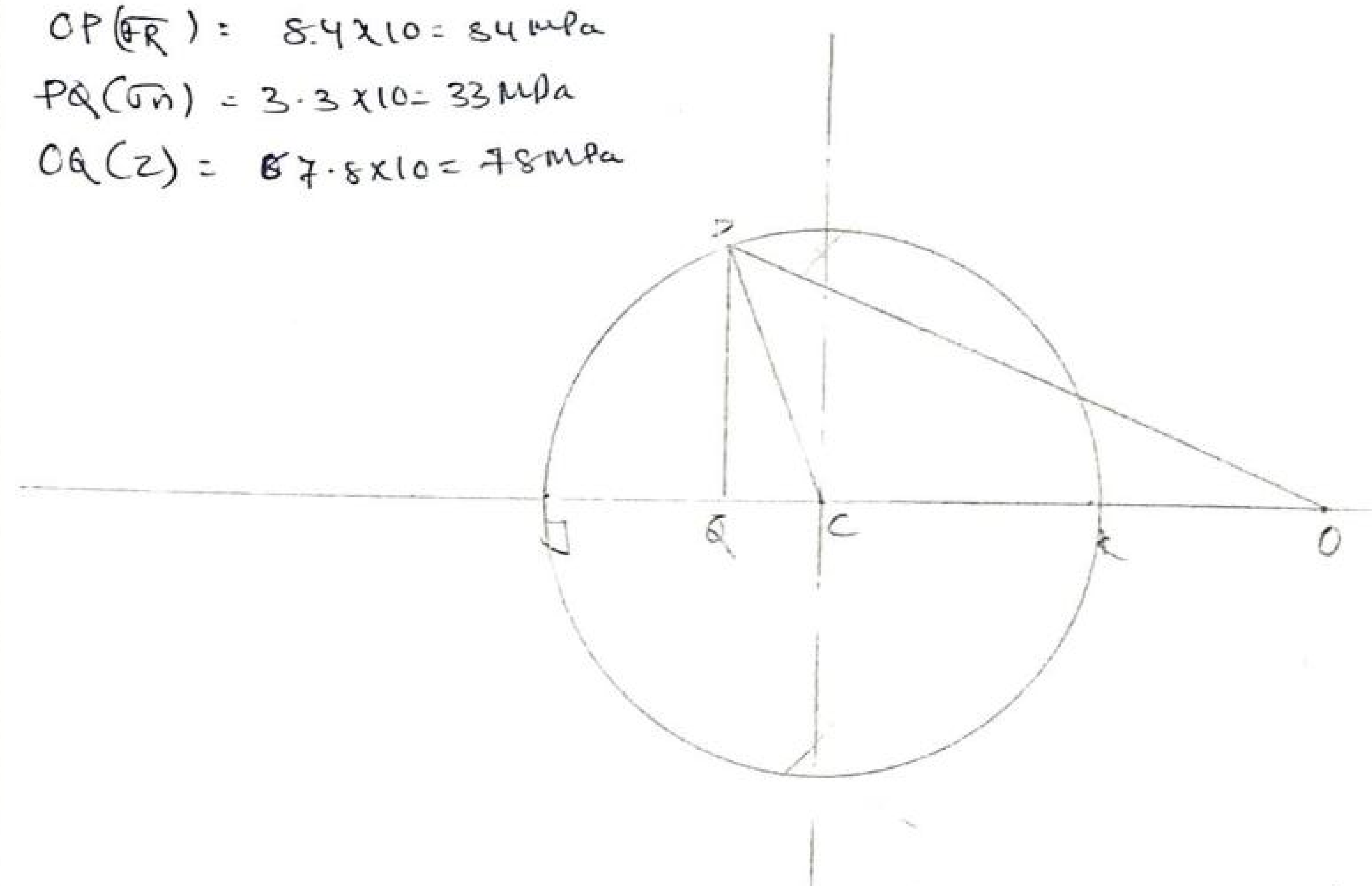
$\sigma_y = 30$ MPa (") $= \frac{30}{10} = 3$ cm.

$\theta = 35^\circ \Rightarrow 2\theta = 2 \times 35^\circ = 70^\circ$

$OP(\sigma_R) = 8.4 \times 10 = 84$ MPa

$PQ(\sigma_n) = 3.3 \times 10 = 33$ MPa

$OQ(\tau) = 7.8 \times 10 = 78$ MPa



Case 3:- stress on an inclined body subjected to one direct stress and one shear stress

1) Take a point O draw a line through it.

2) Cut 'OJ' equal to the direct stress according to the nature of the stress

3) Draw perpendicular 'JD' and 'OE' equal to the shear stress and according to the nature of shear stress.

4) Join ED and mark centre C.

5) Taking C as centre and CD as radius draw the mohr circle.

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6) Draw an angle of 2θ at C in clockwise direction meeting the circle at C.

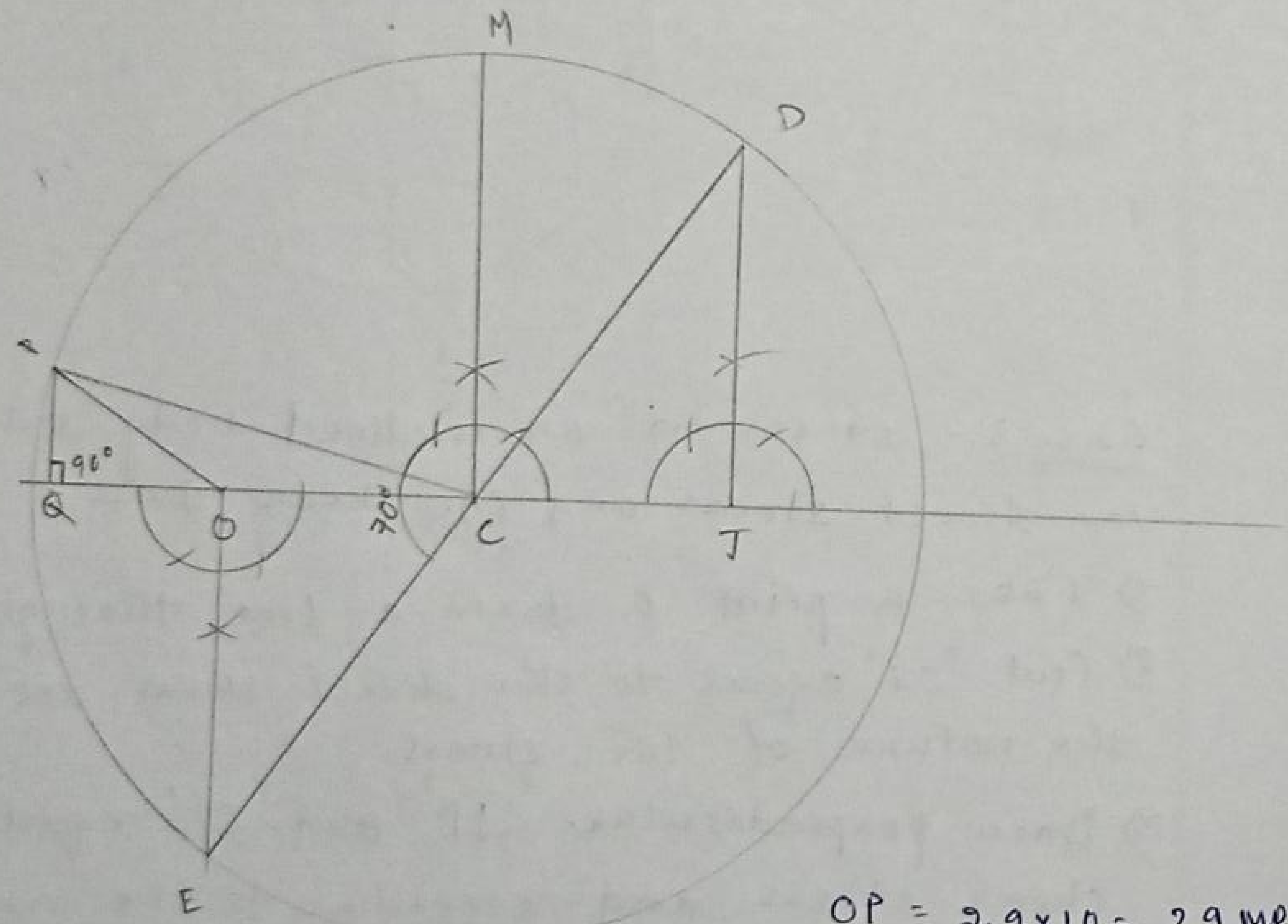
7) Through 'P' draw perpendicular 'PQ' and join 'OP'

8) Measure OQ, PQ, OP, CM.

Q Data given $\sigma = 70\,MPa\,(T) = \frac{70}{10} = 7\,cm$

$Z = 50\,MPa\,(CW) = \frac{50}{10} = 5\,cm$

$\theta = 35^\circ \Rightarrow 2\theta = 70^\circ$

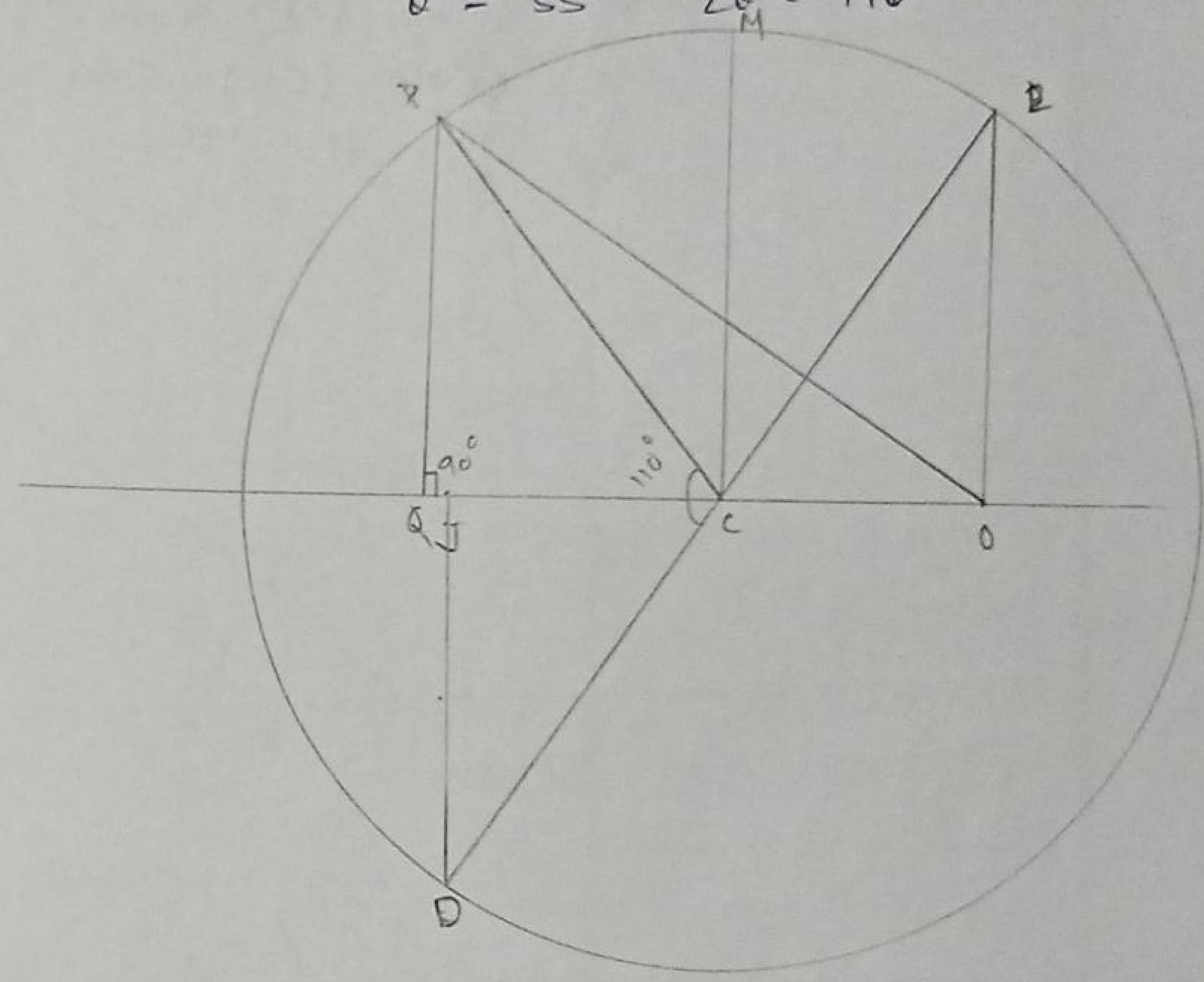


OP = 2.9x10 = 29 MPa
PQ = 1.6x10 = 16 MPa.
OQ = 2.4x10 = 24 MPa
CM = 6.1 x 10 = 61 MPa

Q Data given $\sigma = 80\,MPa\,(C) = 8\,cm$

$Z = 60\,MPa\,(A.C.W) = 6\,cm.$

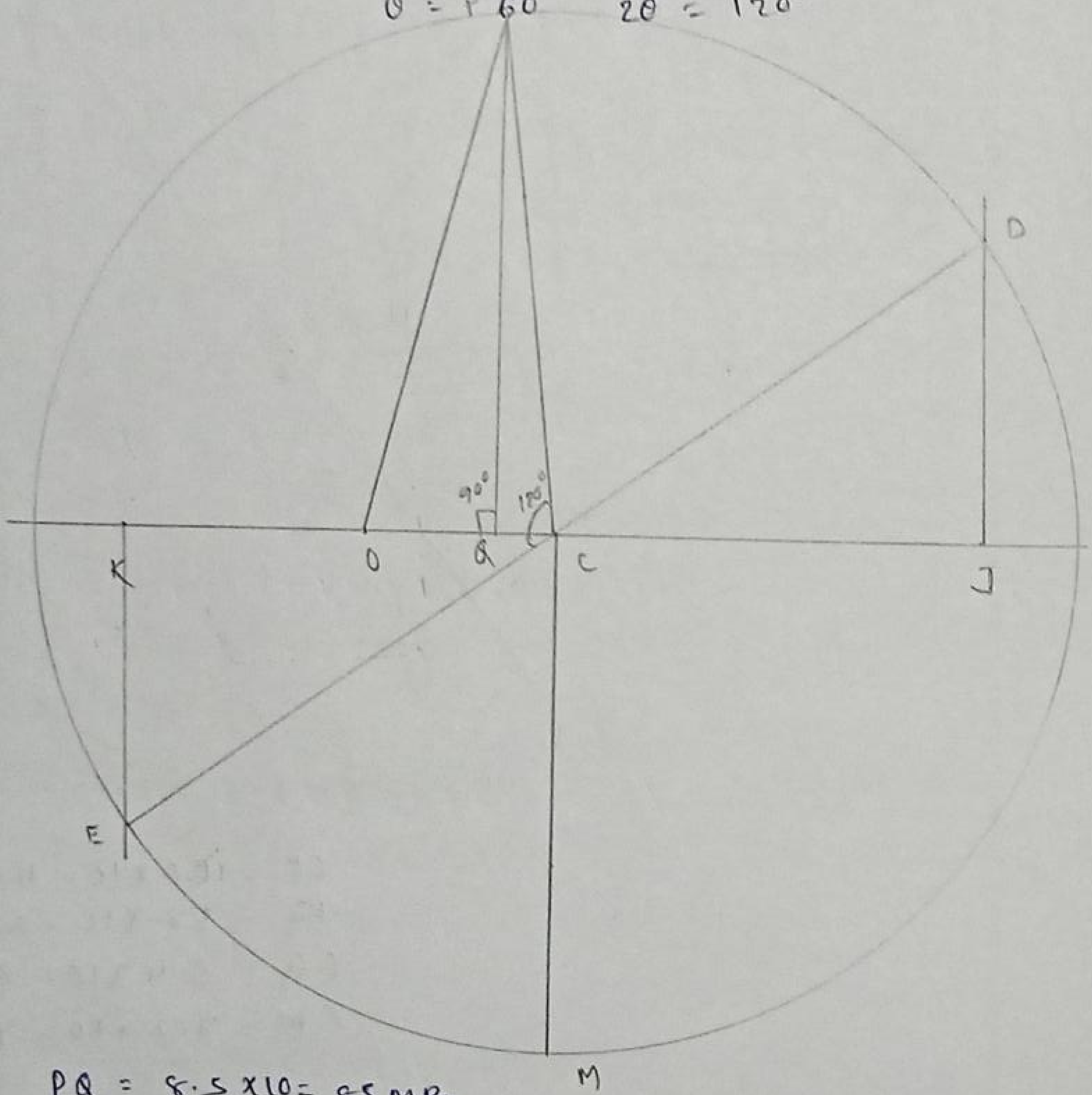
$\theta = 55^\circ \quad 2\theta = 110^\circ$



OP = 10.0 x 10 = 100 MPa.
PQ = 5.8 x 10 = 58 MPa.
OQ = 8.4 x 10 = 84 MPa.
CM = 7.2 x 10 = 72 MPa.

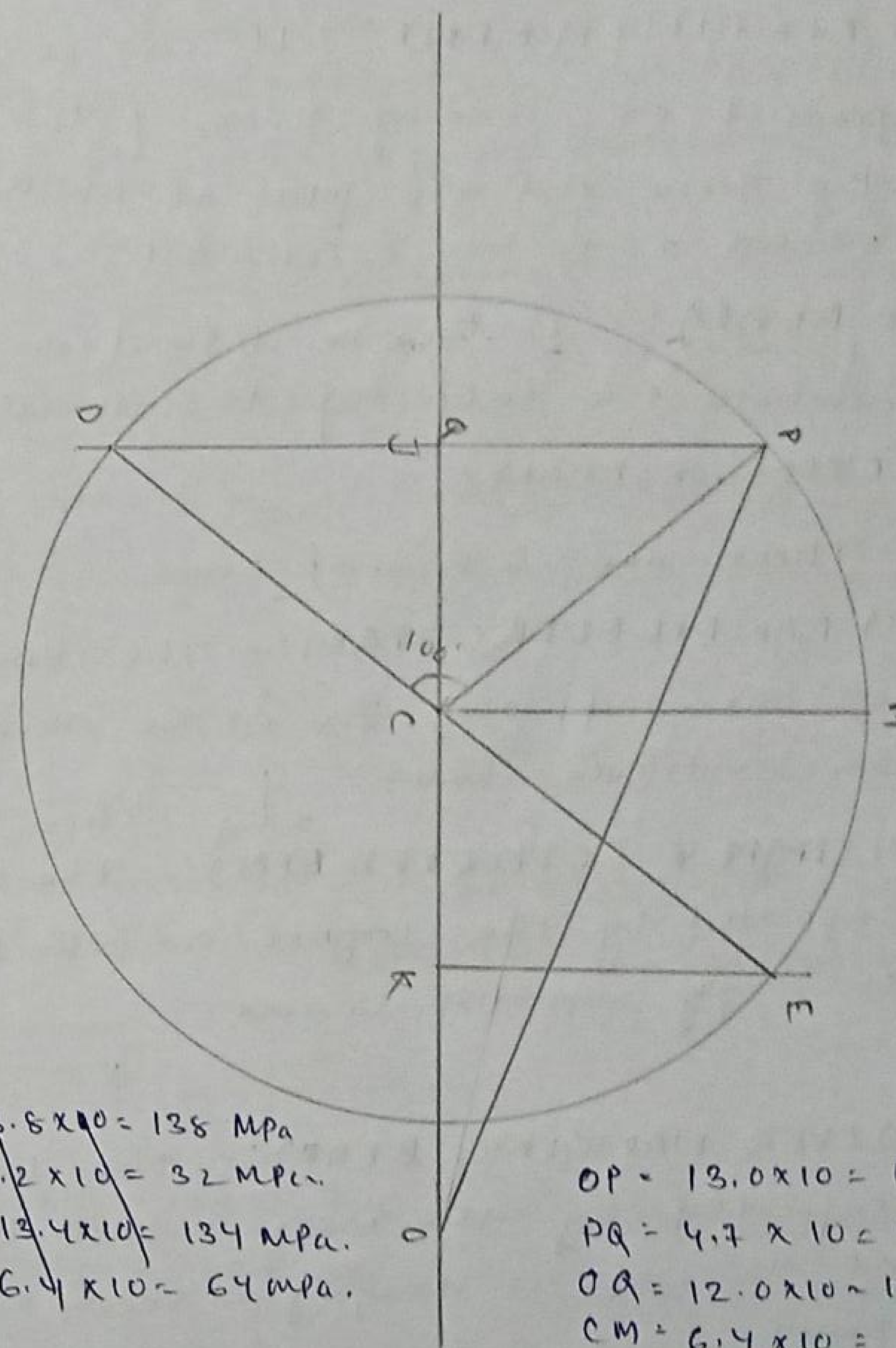
Case 4:-

Q Data given: σ_x = 100 MPa (t) = 10 cm
σ_y = 40 MPa (c) = 4 cm.
τ = 50 MPa (C.W) = 5 cm
θ = 60° 2θ = 120°



PQ = 8.5 × 10 = 85 MPa
OP = 8.9 × 10 = 89 MPa.
OQ = 1.9 × 10 = 19 MPa.
CM = 8.6 × 10 = 86 MPa.

Q Data given:- σ_x = 120 MPa (c) = 12 cm
σ_y = 40 MPa (c) = 4 cm
τ = 50 MPa (A.C.W) = 5 cm
θ = 50° 2θ = 100°



OP = 13.8 × 10 = 138 MPa
PQ = 3.2 × 10 = 32 MPa.
OQ = 13.4 × 10 = 134 MPa.
CM = 6.4 × 10 = 64 MPa.

OP = 13.0 × 10 = 13 MPa.
PQ = 4.7 × 10 = 47 MPa.
OQ = 12.0 × 10 = 12 MPa
CM = 6.4 × 10 = 64 MPa.

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(CH-6) SHEAR FORCE AND BENDING MOMENT

* **SHEAR FORCE** :- The algebric sum of all the force as that are subjected on a beam is known as shear force.

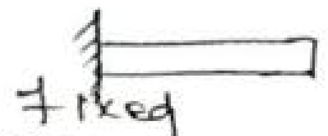
* **BENDING MOMENT** :- It can be defined as the product of bending force & the distance between the force and the point at which the bending moment is to be calculated.

* **BEAM** :- It can be defined as a structural member i.e subjected to external force.

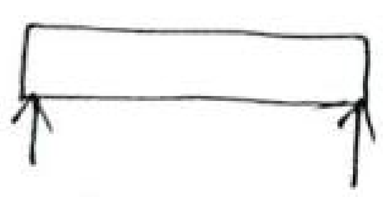
TYPES OF BEAM

There are 5 types of beam.

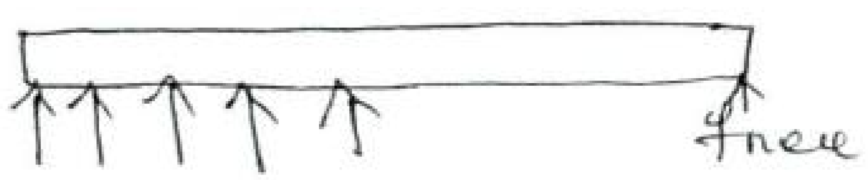
(1) **CANTILEVER BEAM** :- The beam which is free on one end and fix on the other end is known as cantilever beam.



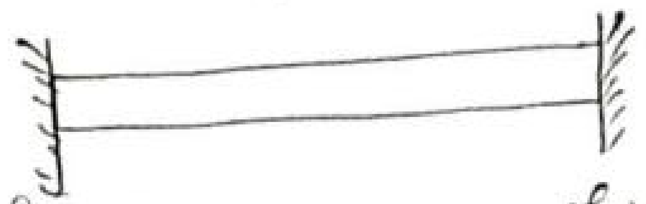
(2) **SIMPLY SUPPORTED BEAM** :- The beam which is supported by the support on both the end is known as simply supported beams.



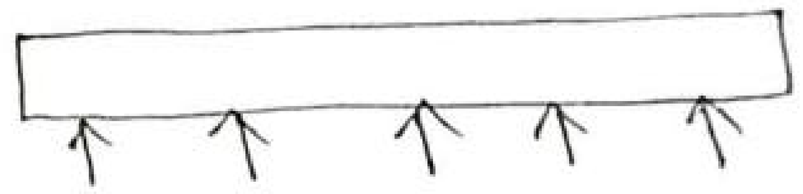
(3) **OVER HANGING BEAM** :- The beam which is supported by the many supports but still one of its ends is hanging is known as over hanging beam.



(4) **FIXED BEAM** :- The beam which is fixed on ~~the~~ both the ends known as fixed beam.



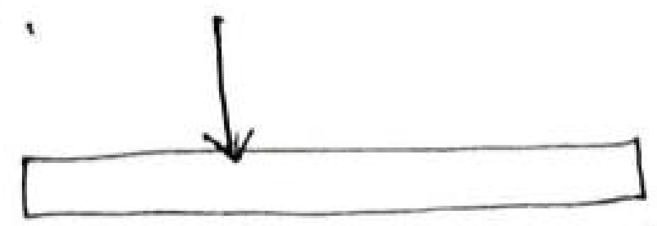
(5) **CONTENIOUS BEAM** :- The beam which is supported by many suppourts throughout it's length.



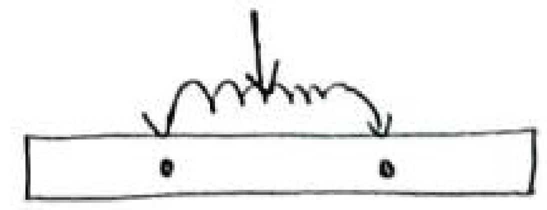
TYPES OF LOADING

There are 3 type of loading subjected on a beam

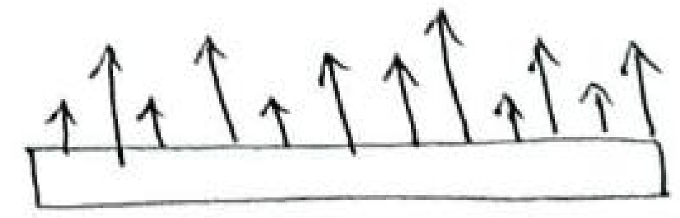
1) Points load ⇒ The load which is concentrated on a point.



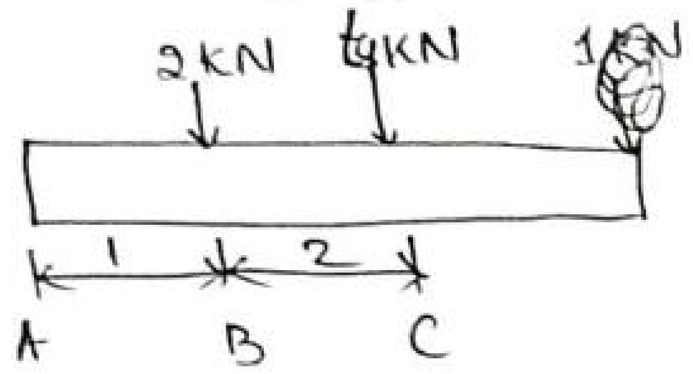
2) Uniform distributed load (UDL) ⇒ The load which is subjected on a particular section of the beam.



3) Varying load ⇒ The load which magnitude keeps changing is known as varying load.



* Bending moments = $F_1 \times D_1 + F_2 D_2 + F_3 D_3$



= ~~4 x 3 + 2 x 1 + 0~~

= ~~12 + 2 = 14 KNM~~

$(B.M)_C = 4 \times 0 = 0$

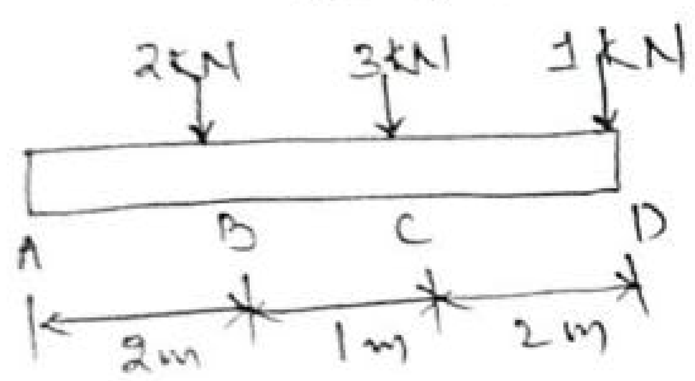
$(B.M)_B = 4 \times 2 + 2 \times 0$

$= 8 + 0 = 8$ KN M

$(B.M)_A = (4 \times 3) + (2 \times 1)$

$= 12 + 2 = 14$ KN

Q



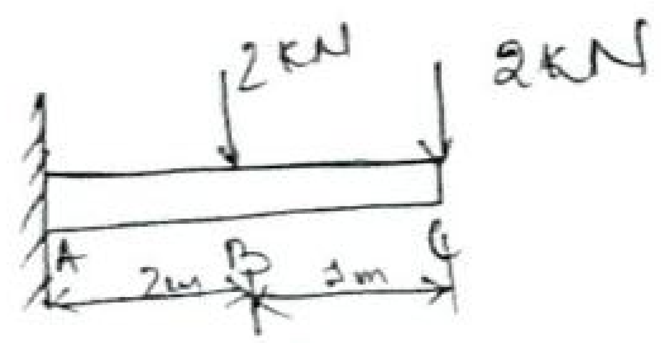
B.M (D) = $1 \times 0 = 0$

B.M (C) = $(1 \times 2) + (3 \times 0) = 2$ KN

B.M (B) = $(x3) + (3 \times 1) + (2 \times 0) = 6$ KN

B.M (A) = $(x5) + (3 \times 3) + (2 \times 2) = 18$ KN

CANTLEVER BEAM (subjected to point load)
(right to left calculation)



Shear force (+)

S.F (C) = +2 KN

S.F (B) = +(2 + 2) = +4 KN

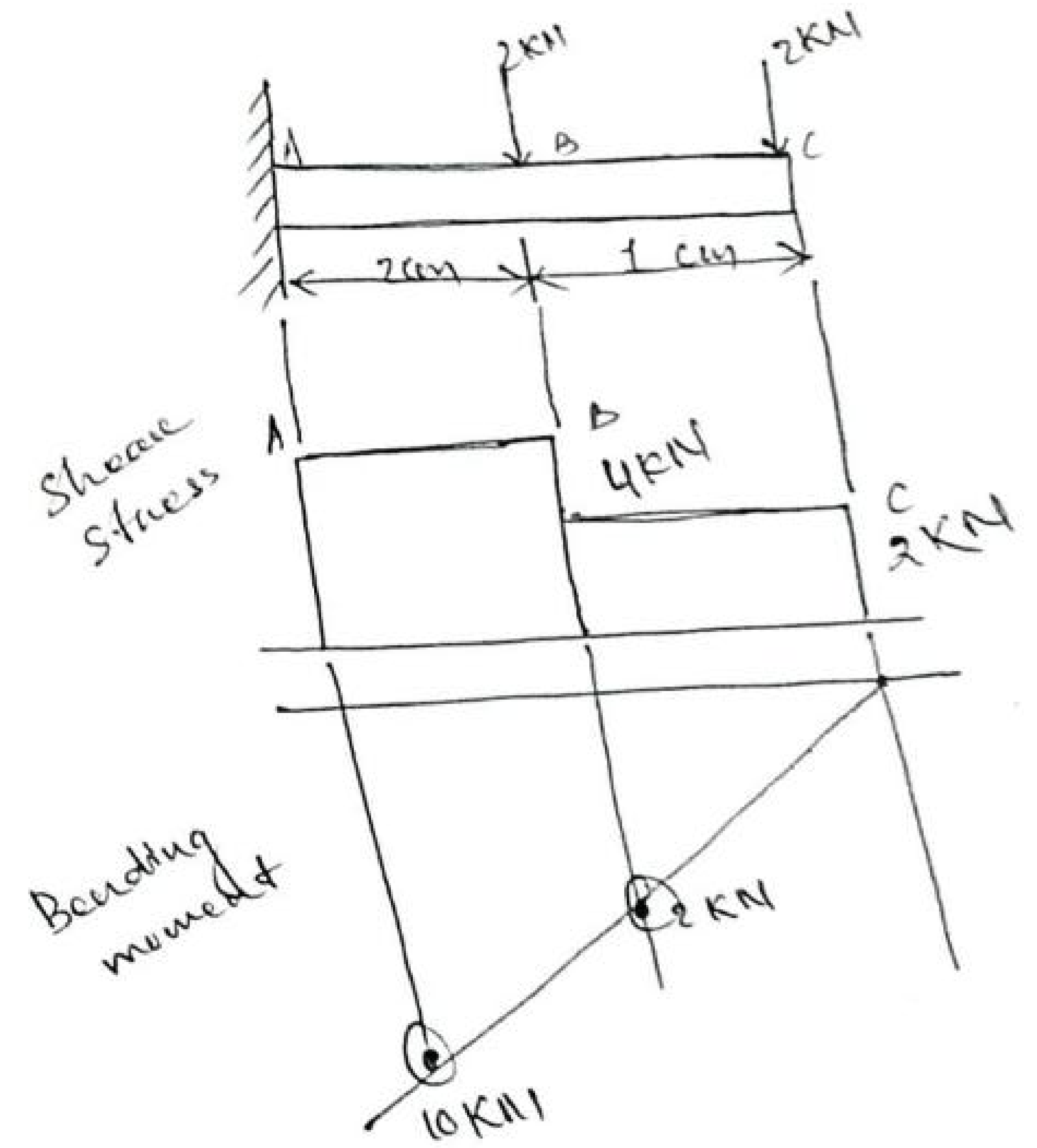
S.F (A) = +(2 + 2) = +4 KN

Bending moments -

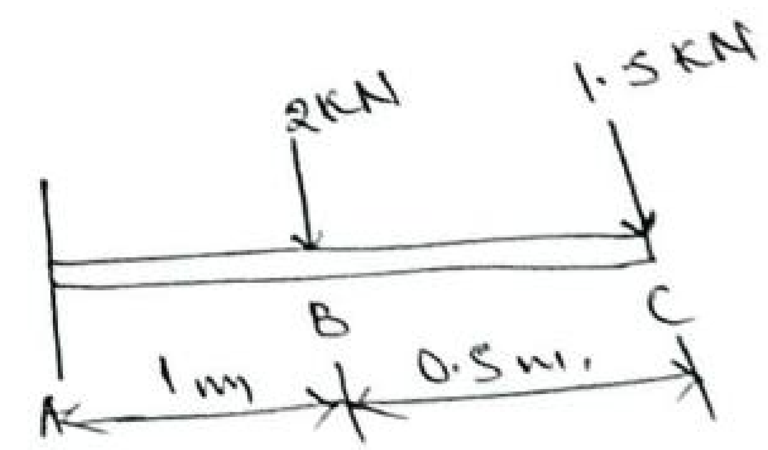
B.M (C) = $-2 \times 0 = 0$

B.M (B) = $-[(2 \times 1) + (2 \times 0)] = -2$ KN

B.M (A) = $-[(2 \times 3) + (2 \times 2)] = -10$ KN



Q



Shear force

S.F (C) = + 1.5 KN

SF (B) = + (1.5 + 2) = 2.5 KN

SF (A) = + (1.5 + 2) = 2.5 KN