<u>**DEPARTMENT OF ELECTRICAL ENGINEERING**</u>

LECTURE NOTES

ON

CONTROL SYSTEM ENGINEERING

6TH SEMESTER

ELECTRICAL ENGINEERING

PREPARED BY

ER. PRITEE PRAVA MINZ

SR. LECTURER ELECTRICAL

GOVERNMENT POLYTECNIC BARGARH

Vision of the Program:

To produce Electrical Engineering professionals who can contribute for socio-economic and technological development to meet global needs.

Mission of the Program:

M1: To Strengthen academic infrastructure leading to quality professional by using modern technical tools and technologies.

M2: To impart innovative knowledge among the students and make more industry-institution programs to make them successful professionals for serving the society.

M3: To Provide a learning environment to improve problem solving abilities, leadership abilities, ethical responsibilities and lifelong learning.

PROGRAM OUTCOMES:

The Electrical diploma engineers will be able to:-

PO1:-Basic and discipline specific knowledge: - Apply knowledge of basic mathematics, science and engineering fundamentals and engineering specializations to solve the engineering problems.

PO2:-Problem Analysis: - Identify and analyze well-defined engineering problems using codified standard methods.

PO3:-Design/development of solutions: - Design solutions for well-defined technical problems and assist with the design of systems components or processes to meet specified needs.

PO4:-Engineering Tools, Experimentation and Testing: - Apply modern engineering tools and appropriate technique to conduct standard tests and measurements.

PO5:-Engineering practices for society, sustainability and environment: - Apply appropriate technology in context of society, sustainability, environment and ethical practices.

PO6:-Project Management: - Use engineering management principles individually, as a team member or a leader to manage projects and effectively communicate about well-defined engineering activities.

PO7:-Life-Long Learning: - Ability to analyze individual needs and engage in updating in the context of technological changes.

PROGRAM SPECIFIC OUTCOMES

The electrical diploma engineers will be able to:

PSO1:- Apply the fundamentals of mathematics, science and engineering knowledge to identify, formulate, design and investigate engineering problems of electric circuits, analog and digital electronics, control system, electrical machines and power system.

PSO2:-Apply the appropriate techniques and modern engineering hardware and software tools in electrical engineering to engage in life-long learning and to successfully adapt in multi-disciplinary environments.

PSO3:-Work professionally in power systems engineering, Electrical machinery and electrical circuits. Process Diagram to Identify Extent of Compliance of the SCTE&VT Curriculum for Attaining the Program Outcomes and Program Specific Outcomes.

Program Educational Objectives (PEOs)

PEO1 -To obtain basic and advanced knowledge in Electrical Engineering for employment in public/private sector organizations.

PEO2 - To encourage the students for higher studies by acquiring knowledge in the basic and emerging areas of Electrical Engineering.

PEO3 -To become entrepreneurs to showcase innovative ideas.

PEO4 - To have a well-rounded education that includes excellent communication skills, working effectively on team-based projects, ethical and social responsibilities.

CIRCUIT & NETWORK THEORY

CO1:-Compare between magnetic circuit and couple circuit.

CO2:- Understand the concept of Network analysis and Theorems and apply in solving problems.

CO3:- Explain and analyze the concept of RL, RC, and RLC circuit.

CO4:- Explain the concept of poly-phase circuit to solve numerical problems.

CO5:- Analyze the electrical circuit parameters and construct different type of filters.

Syllabus

Th3.CONTROL SYSTEM ENGINEERING

Name of the Course: Diploma in Electrical Engineering			
Course code:		Semester	6^{th}
Total Period:	75	Examination	3 hrs
Theory periods:	4 P / week	Class Test:	20
Tutorial:	1 P / week	End Semester Examination:	80
Maximum marks:	100		

A. RATIONALE:

Automatic control has played a vital role in modern Engineering and Science. It has become an indispensable part of modern manufacturing and industrial process. So knowledge of automatic control system is dreadfully essential on the part of an Engineer. Basic approach to the automatic control system has been given in the subjects, so that students can enhance their knowledge in their future professional carrier.

B. OBJECTIVE:

Study of 'Control System' enhances the ability of the student on:

1. Acquire knowledge about Mathematical modeling, Block diagram algebra, signal flow graphs and control system components.
2. Ability to deal with time response analysis of various systems.
3. Finding out steady state error and error constants.
4. Acquire knowledge about the analysis of stability in Root locus technique.
5. Learning about frequency response analysis of control system.
6. To use Bode plot and Nyquist plot for judgments about stability of a system.

C. Topic wise distribution of periods:

Sl. No.	Topics	Periods
1.	Fundamental of control system	04
2.	Mathematical model of a system	04
3.	Control system components	04
4.	Block diagram algebra & signal flow graphs	08
5.	Time response analysis	10
6.	Analysis of stability by root locus technique	10
7.	Frequency response of system	10
8.	Nyquist plot	10
	Total	**60**

D. COURSE CONTENTS

1. FUNDAMENTAL OF CONTROL SYSTEM

1.1. Classification of Control system
1.2. Open loop system & Closed loop system and its comparison
1.3. Effects of Feed back
1.4. Standard test Signals(Step, Ramp, Parabolic, Impulse Functions)
1.5. Servomechanism

2. MATHEMATICAL MODEL OF A SYSTEM

2.1. Transfer Function & Impulse response,
2.2. Properties, Advantages & Disadvantages of Transfer Function
2.3. Poles & Zeroes of transfer Function
2.4. Simple problems of transfer function of network.
2.5. Mathematical modeling of Electrical Systems(R, L, C, Analogous systems)

3. CONTROL SYSTEM COMPONENTS

3.1. Components of Control System
3.2. Gyroscope, Synchros, Tachometer, DC servomotors, Ac Servomotors.

4. BLOCK DIAGRAM ALGEBRA & SIGNAL FLOW GRAPHS

4.1. Definition: Basic Elements of Block Diagram
4.2. Canonical Form of Closed loop Systems
4.3. Rules for Block diagram reduction
4.4. Procedure for of Reduction of Block Diagram
4.5. Simple Problem for equivalent transfer function
4.6. Basic Definition in Signal Flow Graph & properties
4.7. Construction of Signal Flow graph from Block diagram
4.8. Mason's Gain formula
4.9. Simple problems in Signal flow graph for network

5. TIME RESPONSE ANALYSIS.

5 . 1 Time response of control system.
5 . 2 Standard Test signal.
 5.2.1. Step signal,
 5.2.2. Ramp Signal
 5.2.3. Parabolic Signal
 5.2.4. Impulse Signal
5 . 3 Time Response of first order system with:
 5.3.1. Unit step response
 5.3.2. Unit impulse response.
5 . 4 Time response of second order system to the unit step input.
 5.4.1. Time response specification.
 5.4.2. Derivation of expression for rise time, peak time, peak overshoot, settling time and steady state error.

5.4.3. Steady state error and error constants.

5 . 5 Types of control system.[Steady state errors in Type-0, Type-1, Type-2 system]

5 . 6 Effect of adding poles and zero to transfer function.

5 . 7 Response with P, PI, PD and PID controller.

6. ANALYSIS OF STABILITY BY ROOT LOCUS TECHNIQUE.

6 . 1 Root locus concept.

6 . 2 Construction of root loci.

6 . 3 Rules for construction of the root locus.

6 . 4 Effect of adding poles and zeros to G(s) and H(s).

7. FREQUENCY RESPONSE ANALYSIS.

7 . 1 Correlation between time response and frequency response.

7 . 2 Polar plots.

7 . 3 Bode plots.

7 . 4 All pass and minimum phase system.

7 . 5 Computation of Gain margin and phase margin.

7 . 6 Log magnitude versus phase plot.

7 . 7 Closed loop frequency response.

8. NYQUIST PLOT

8.1 Principle of argument.

8.2 Nyquist stability criterion.

8.3 Niquist stability criterion applied to inverse polar plot.

8.4 Effect of addition of poles and zeros to G(S) H(S) on the shape of Niquist plot.

8.5 Assessment of relative stability.

8.6 Constant M and N circle

8.7 Nicholas chart.

Syllabus coverage up to Internal assessment

Chapters: 1, 2, 3, 4 and 5.

Learning Resources:			
Sl.No	**Title of the Book**	**Name of Authors**	**Name of Publisher**
1.	Control System	A. Ananda Kumar	PHI
3.	Control System	K. Padmanavan	IK
2.	Control system Engineering	I. J. Nagarath, M. Gopal	WEN
4.	Control system Engineering	A Natrajan,Ramesh Babu	Scientific
5.	Control Systems	D N Manik	Cengage
6.	Control Systems	S P Eugene Xavier, J Joseph Cyril Babu	S Chand

GOVT. POLYTECHNIC, BARGARH
AT-TENTLA, PO-KATAPALI, VIA-BARDOL, BARGARH-768038

VISION

To be a reputed polytechnic institute imparting quality technical education to produce diploma engineers with dynamic personalities and innovative competencies in the state of Odisha.

MISSION

M1:- To offer the best and advanced lab facilities adhering to the curriculum to make future engineers.

M2:- To engage highly qualified and competent faculties to make the student acquire the skilful knowledge required.

M3:- To develop an excellent teaching learning environment leading to create the best institute.

PROGRAM OUTCOMES (Pos)

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DEPT OF ELECTRICAL ENGINEERING, G.P. BARGARH

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PSO2: - Apply the appropriate techniques and modern engineering hardware and software tools in electrical engineering to engage in life-long learning and to successfully adapt in multi-disciplinary environments.

PSO3: -Work professionally in power systems engineering, Electrical machinery and electrical circuits.

COURSE CODE : TH3(CSE)

SEMESTER : 6TH

At the end of the course, a student will be able to:

CO1. Categorize different types of control system and standard test signal.

CO2. Understand property of transfer function and its components.

CO3: Determine mathematical modeling of an electrical system

CO 4: Characterize any system in Laplace domain to illustrate different specification of the system using transfer function concept by using block diagram reduction and signal flow graph.

CO5. Employ time domain analysis to predict and diagnose transient performance parameters of the system for standard input functions. Identify the needs of different types of controllers.

CO6. Formulate different types of analysis in time and frequency domain to explain the nature of stability of the system

CO-PO-PSO MAPPING										
CO	PO1	PO2	PO3	PO 4	PO 5	PO 6	PO 7	PS O1	PS O2	PSO 3
CO.1	3	3	1	-	-	-	-	3	-	-
CO.2	3	3	1	-	-	-	-	3	-	-
CO.3	3	3	2	-	-	-	-	3	-	-
CO.4	3	3	2	-	-	-	-	3	-	-
CO.5	3	3	1	-	1	2	-	3	-	-
CO.6	3	3	2	-	1	2	-	3	-	-
Average	3	3	1.5	-	1	2	-	3	-	-

Ch-1
TRANSFER FUNCTION

Introduction:

Transfer function of a linear time-invariant system is defined as the ratio of the Laplace transform of output variable to the Laplace transform of input variable assuming all the initial conditions to be zero. The figure 1a shows the system in time domain whereas figure 1b shows the system in Laplace domain.

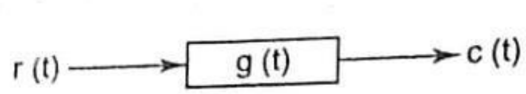


Figure 1a. system in time domain

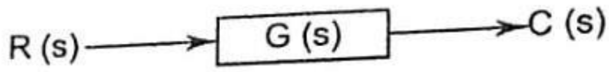


Fig1b. system in Laplace domain.

Figure 1. Transfer Function of a system

If G(s) be the transfer function of the system, we can write mathematically as

$G(s) = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}}$ (all initial conditions are zero)

$= \frac{C(s)}{R(s)}$(all initial conditions are zero) ....(1)

Example: Determine the transfer function of figure 2 shown below. $V_i(t)$ is the input to the system and $V_o(t)$ is the output of the system.

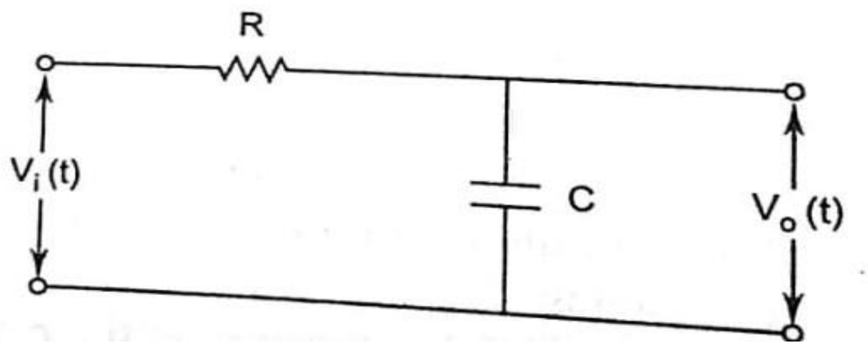


Figure 2

Solution- Let i(t) be the current flowing through the circuit using KVL we can write

$$V_i(t) = Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

$$\text{And } V_o(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

Taking Laplace transfer of the above equation by assuming zero initial condition, we get $V_i(s)$

$$= RI(s) + \frac{1}{sC} I(s)$$

$$\text{And } V_o(s) = \frac{1}{sC} I(s)$$

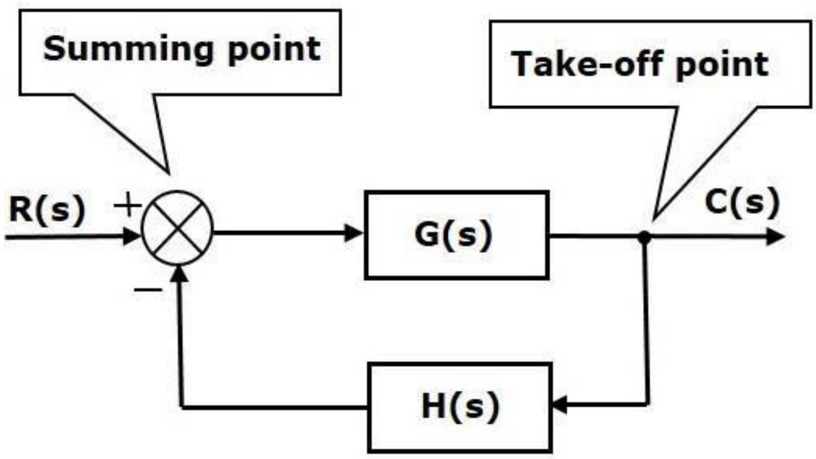
$$\therefore Transfer\ Function = G(s) = \frac{Vo(s)}{Vi(s)} = \frac{1}{1+sCR}$$

BLOCK DIAGRAMS

Block diagram is the pictorial representation of system. It consists of a single block or a combination of blocks. Each block is a functional block.

Basic Elements of Block Diagram

The basic elements of a block diagram are a block, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following figure to identify these elements.



The above block diagram consists of two blocks having transfer functions G(s) and H(s). It is also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals. Let us now discuss these elements one by one.

Block

The transfer function of a component is represented by a block. Block has single input and single output.

The following figure shows a block having input X(s), output Y(s) and the transfer function G(s).

Transfer Function, $G(s)=Y(s)/X(s)$

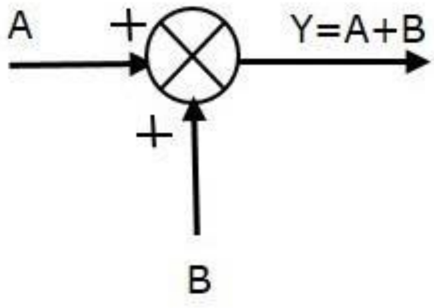
$$\Rightarrow Y(s)=G(s)X(s)$$

Output of the block is obtained by multiplying transfer function of the block with input.

Summing Point

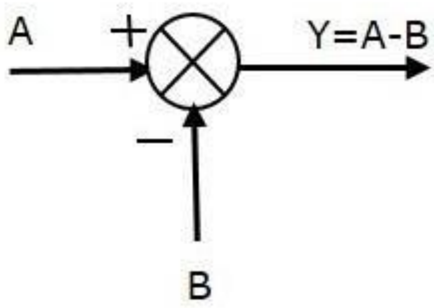
The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs. Let us see these three operations one by one.

The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y as **sum of A and B.** i.e.,Y = A + B.

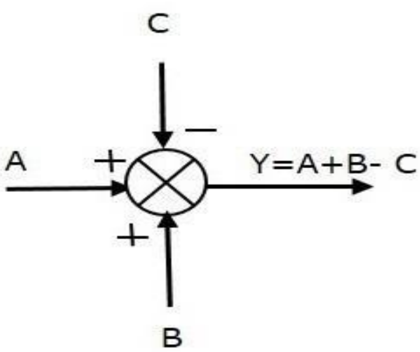


The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B are having opposite signs, i.e., A is having positive sign and B is having negative sign. So, the summing point produces the output **Y** as the **difference of A and B**.

Y = A + (-B) = A - B.



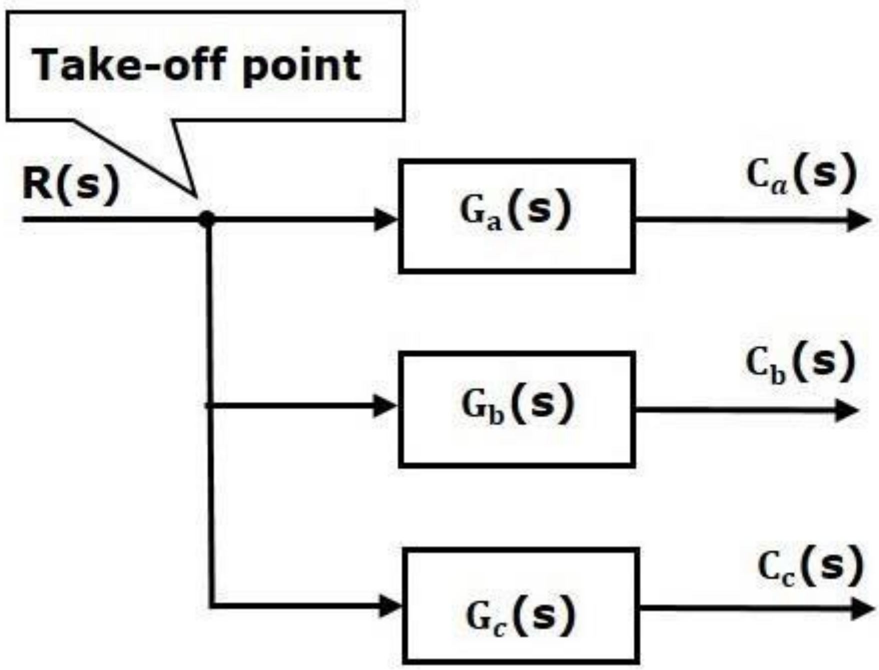
The following figure shows the summing point with three inputs (A, B, C) and one output (Y). Here, the inputs A and B are having positive signs and C is having a negative sign. So, the summing point produces the output **Y** as Y = A + B + (−C) = A + B − C.



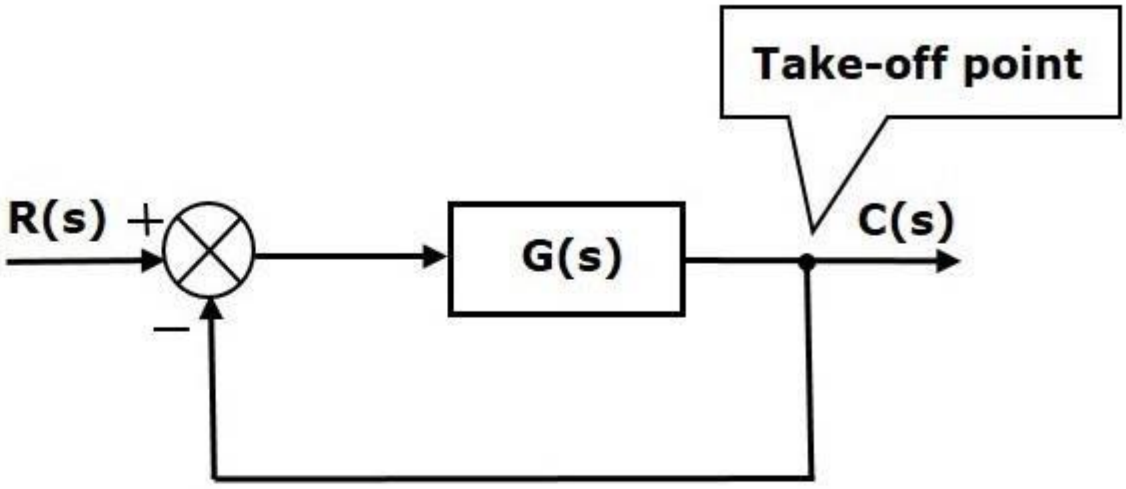
Take-off Point

The take-off point is a point from which the same input signal can be passed through more than one branch. That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.

In the following figure, the take-off point is used to connect the same input, R(s) to two more blocks.



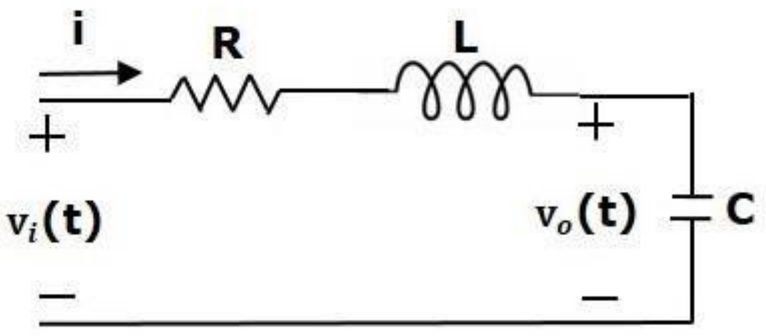
In the following figure, the take-off point is used to connect the output C(s), as one of the inputs to the summing point.



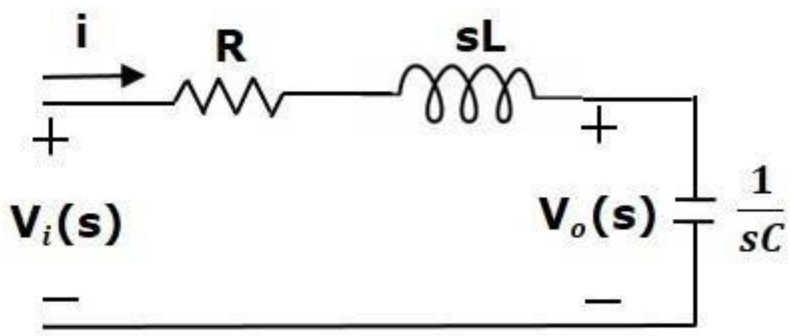
Block Diagram Representation of Electrical Systems

In this section, let us represent an electrical system with a block diagram. Electrical systems contain mainly three basic elements — **resistor, inductor and capacitor**.

Consider a series of RLC circuit as shown in the following figure. Where, $V_i(t)$ and $V_o(t)$ are the input and output voltages. Let i(t) be the current passing through the circuit. This circuit is in time domain.



By applying the Laplace transform to this circuit, will get the circuit in s-domain. The circuit is as shown in the following figure.



From the above circuit, we can write

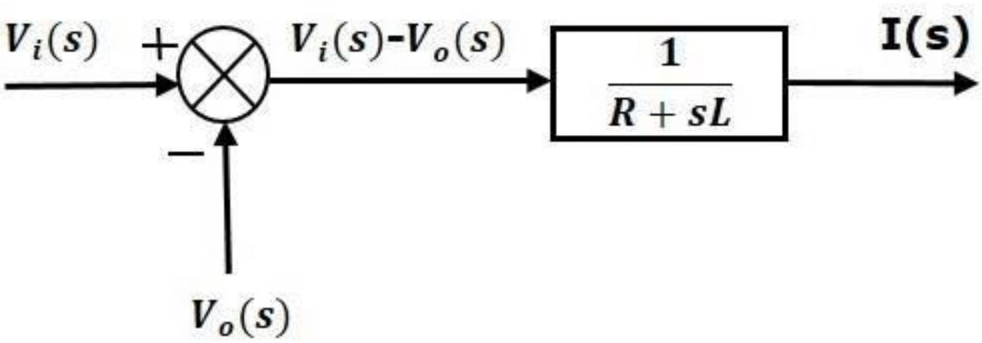
$$I(s)=[Vi(s)-Vo(s)]/R+sL$$

$\Rightarrow I(s)=\{1/R+sL\}\{Vi(s)-Vo(s)\}$ **(Equation 1)**

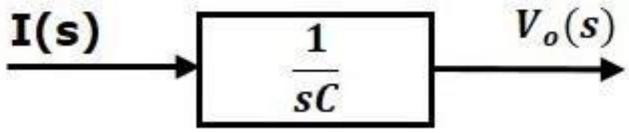
$Vo(s)=(1/sC)I(s)$ **(Equation 2)**

Let us now draw the block diagrams for these two equations individually. And then combine those block diagrams properly in order to get the overall block diagram of series of RLC Circuit (s-domain).

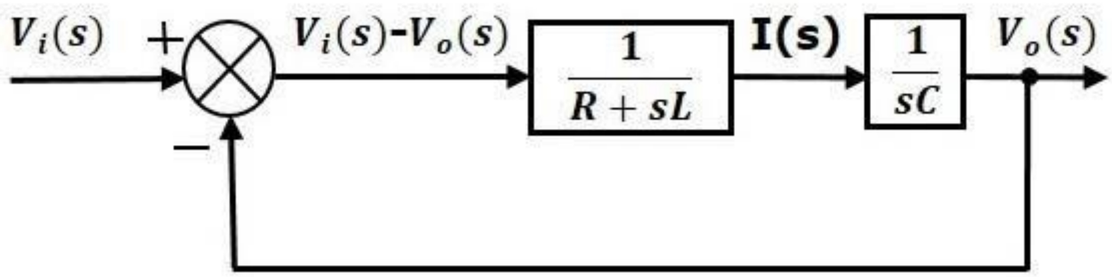
Equation 1 can be implemented with a block having the transfer function, 1/R+sL. The input and output of this block are {Vi(s)−Vo(s)} and I(s). We require a summing point to get {Vi(s)−Vo(s)}. The block diagram of Equation 1 is shown in the following figure.



Equation 2 can be implemented with a block having transfer function, 1/sC. The input and output of this block are I(s) and Vo(s). The block diagram of Equation 2 is shown in the following figure.



The overall block diagram of the series of RLC Circuit (s-domain) is shown in the following figure.



Similarly, you can draw the **block diagram** of any electrical circuit or system just by following this simple procedure.

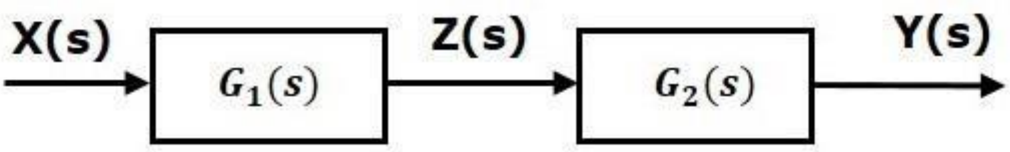
- Convert the time domain electrical circuit into an s-domain electrical circuit by applying Laplace transform.
- Write down the equations for the current passing through all series branch elements and voltage across all shunt branches.
- Draw the block diagrams for all the above equations individually.
- Combine all these block diagrams properly in order to get the overall block diagram of the electrical circuit (s-domain).

Block diagram reduction rules:

There are three basic types of connections between two blocks.

Rule 1: Series Connection

Series connection is also called **cascade connection**. In the following figure, two blocks having transfer functions G1(s) and G2(s) are connected in series.



For this combination, we will get the output Y(s) as

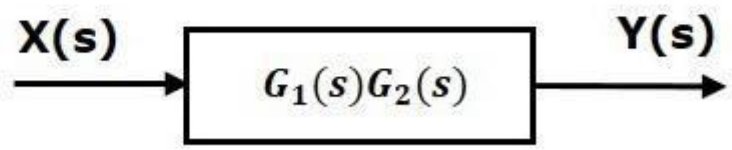
$$Y(s) = G2(s)\ Z(s)$$

Where, Z(s) = G1(s) X(s)

⇒Y(s) = G2(s) [G1(s) X(s)] = G1(s) G2(s) X(s) ⇒ Y(s) = {G1(s) G2(s)} X(s)

Compare this equation with the standard form of the output equation, Y(s) = G(s) X(s). Where, G(s) = G1(s) G2(s).

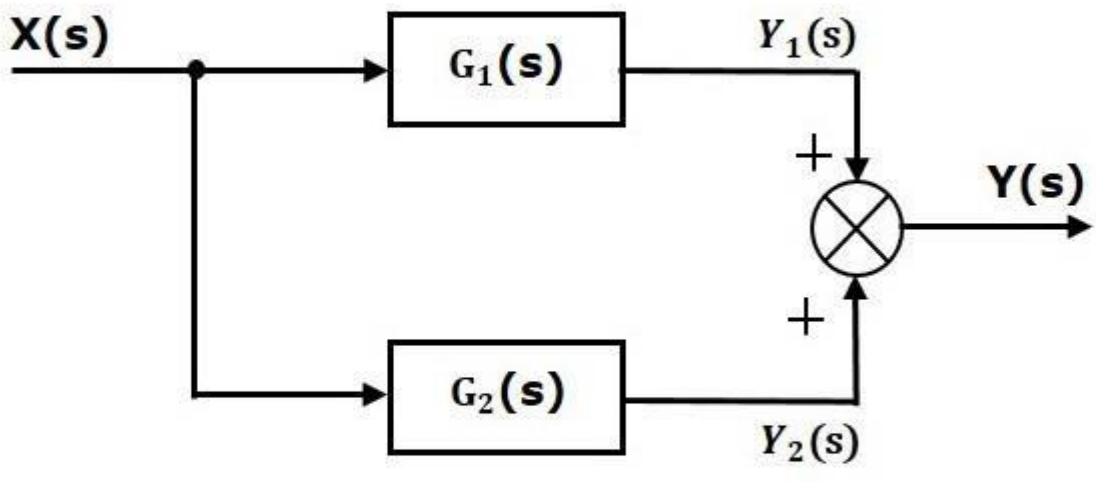
That means we can represent the **series connection** of two blocks with a single block. The transfer function of this single block is the **product of the transfer functions** of those two blocks. The equivalent block diagram is shown below.



Similarly, you can represent series connection of 'n' blocks with a single block. The transfer function of this single block is the product of the transfer functions of all those 'n' blocks.

Rule 2: Parallel Connection

The blocks which are connected in **parallel** will have the **same input**. In the following figure, two blocks having transfer functions G1(s) and G2(s) are connected in parallel. The outputs of these two blocks are connected to the summing point.



For this combination, we will get the output Y(s) as

Y(s) = Y1(s) + Y2(s)

Where, Y1(s) = G1(s) X(s)) and Y2(s) = G2(s) X(s)

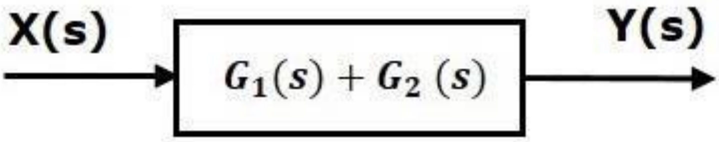
⇒Y(s) = G1(s) X(s) + G2(s) X(s) = {G1(s) + G2(s)} X(s)

Compare this equation with the standard form of the output equation,

Y(s) = G(s) X(s)

Where, G(s) = G1(s) + G2(s)

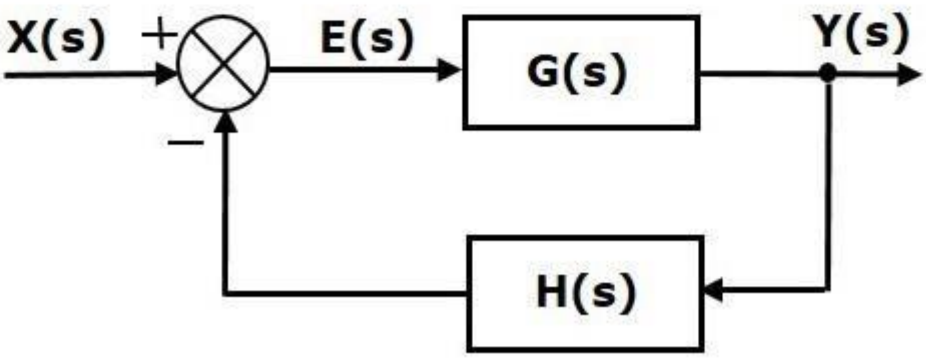
That means we can represent the **parallel connection** of two blocks with a single block. The transfer function of this single block is the **sum of the transfer functions** of those two blocks. The equivalent block diagram is shown below.



Similarly, you can represent parallel connection of ‘n’ blocks with a single block. The transfer function of this single block is the algebraic sum of the transfer functions of all those ‘n’ blocks.

Rule 3: Feedback Connection

As we discussed in previous chapters, there are two types of **feedback** — positive feedback and negative feedback. The following figure shows negative feedback control system. Here, two blocks having transfer functions G(s) and H(s) form a closed loop.



The output of the summing point is -

$$E(s) = X(s) - H(s)\,Y(s)$$

The output Y(s) is -

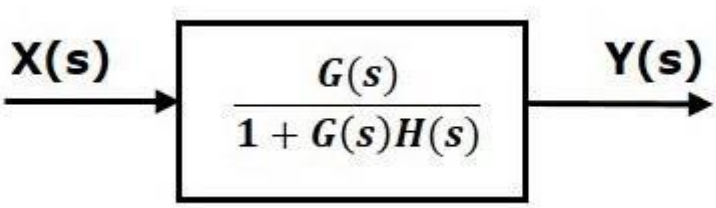
$$Y(s) = E(s)\,G(s)$$

Substitute E(s) value in the above equation.

$Y(s) = \{X(s) - H(s)Y(s)\}\ G(s)\}\ Y(s)\ \{1 + G(s)\ H(s)\} = X(s)\ G(s) \Rightarrow Y(s)/X(s) = G(s) / [1 + G(s)\ H(s)]$

Therefore, the negative feedback closed loop transfer function is G(s) / [1+G(s) H(s)]

This means we can represent the negative feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the negative feedback. The equivalent block diagram is shown below.



Similarly, you can represent the positive feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the positive feedback, i.e., $G(s) / [1 - G(s) H(s)]$

Rule 4: Block Diagram Algebra for Summing Points

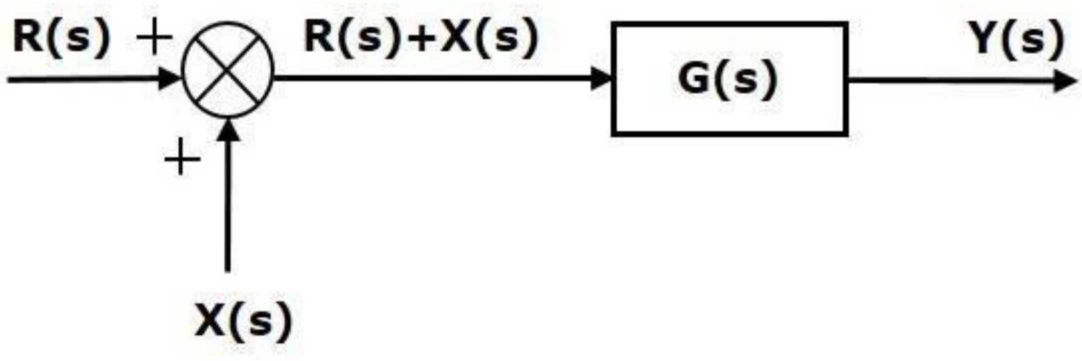
There are two possibilities of shifting summing points with respect to blocks −

- Shifting summing point after the block
- Shifting summing point before the block

Let us now see what kind of arrangements need to be done in the above two cases one by one.

Rule 4a: Shifting Summing Point after the Block

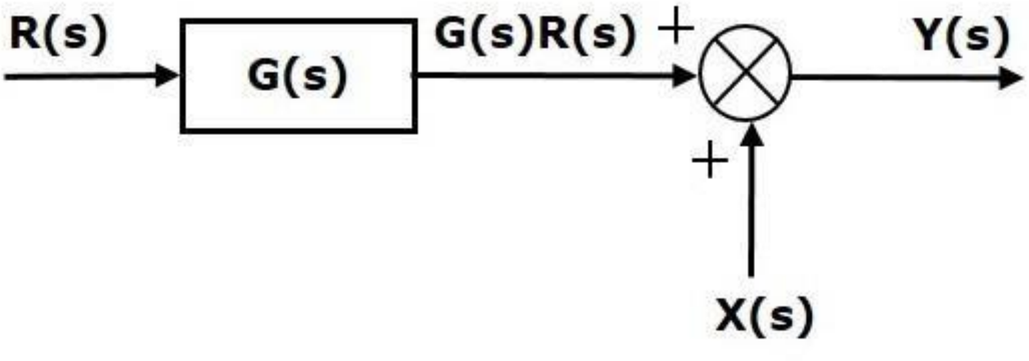
Consider the block diagram shown in the following figure. Here, the summing point is present before the block.



Summing point has two inputs R(s) and X(s). The output of it is {R(s)+X(s)}
So, the input to the block G(s) is {R(s)+X(s)} and the output of it is –

$$Y(s) = G(s)\{R(s)+X(s)\} \Rightarrow Y(s) = G(s)R(s) + G(s)X(s) \quad \textbf{(Equation 1)}$$

Now, shift the summing point after the block. This block diagram is shown in the following figure.



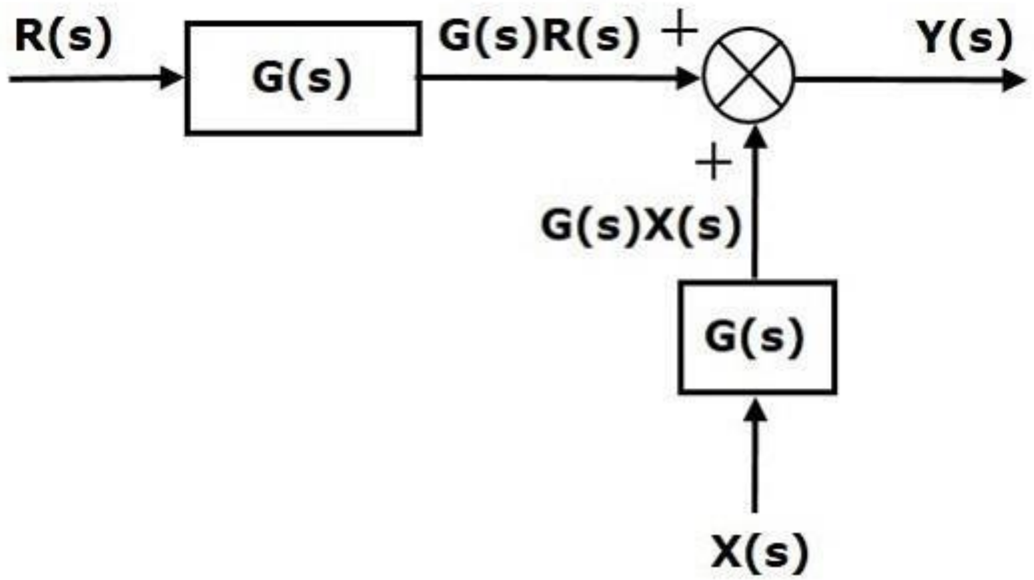
Output of the block G(s) is G(s)R(s)

The output of the summing point is

$Y(s)=G(s)R(s)+X(s)$ **(Equation 2)**

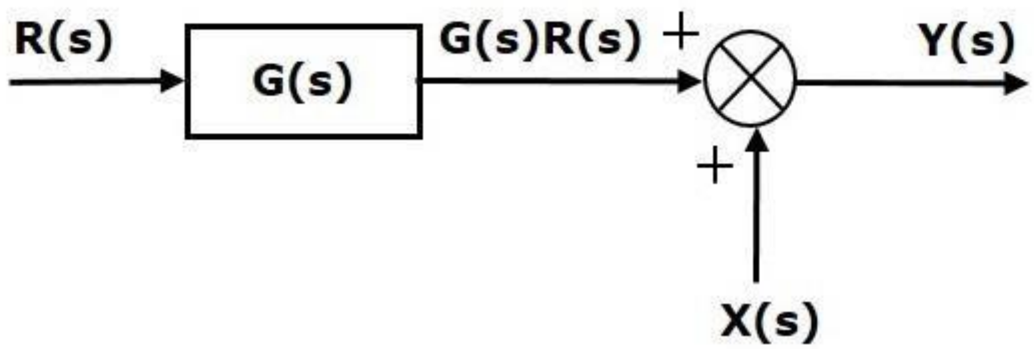
Compare Equation 1 and Equation 2.

The first term 'G(s)R(s)' is same in both the equations. But, there is difference in the second term. In order to get the second term also same, we require one more block G(s). It is having the input X(s) and the output of this block is given as input to summing point instead of X(s).
This block diagram is shown in the following figure.



Rule 4b: Shifting Summing Point Before the Block

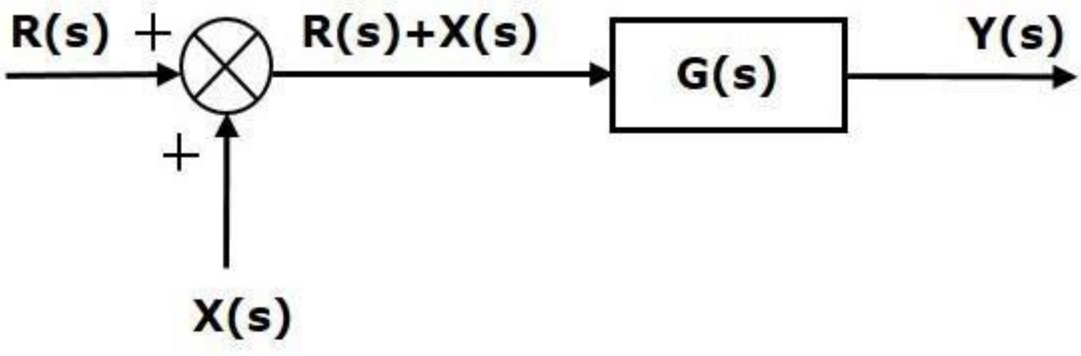
Consider the block diagram shown in the following figure. Here, the summing point is present after the block.



Output of this block diagram is -

Y(s) = G(s) R(s) + X(s) **(Equation 3)**

Now, shift the summing point before the block. This block diagram is shown in the following figure.

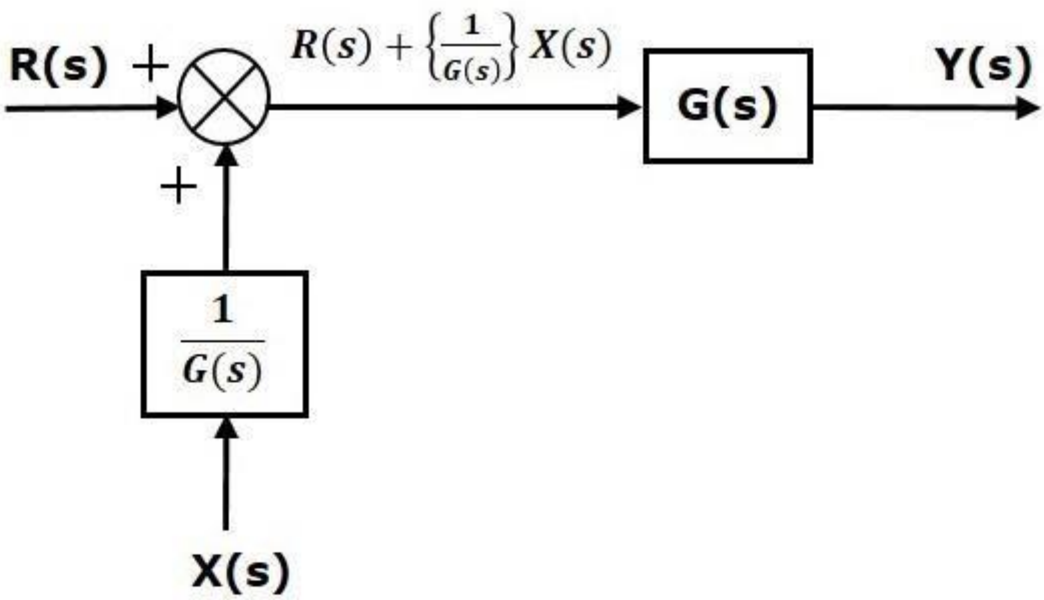


Output of this block diagram is -

Y(S) = G(s) R(s) + G(s) X(s) **(Equation 4)**

Compare Equation 3 and Equation 4,

The first term 'G(s) R(s)' is same in both equations. But, there is difference in the second term. In order to get the second term also same, we require one more block 1/G(s). It is having the input X(s) and the output of this block is given as input to summing point instead of X(s). This block diagram is shown in the following figure.



Rule 5: Block Diagram Algebra for Take-off Points

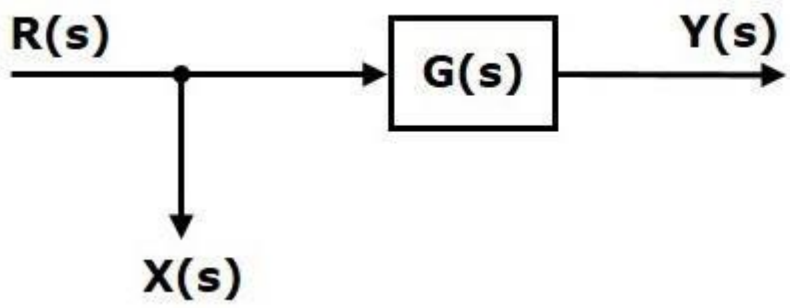
There are two possibilities of shifting the take-off points with respect to blocks −

- Shifting take-off point after the block
- Shifting take-off point before the block

Let us now see what kind of arrangements are to be done in the above two cases, one by one.

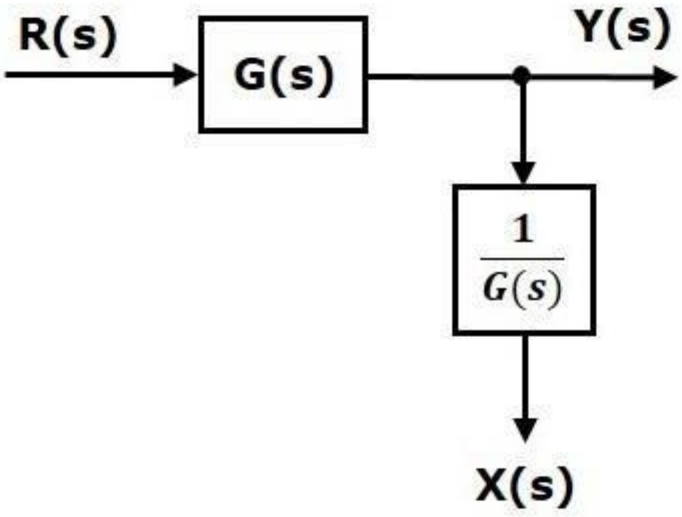
Rule5a: Shifting Take-off Point after the Block

Consider the block diagram shown in the following figure. In this case, the take-off point is present before the block.



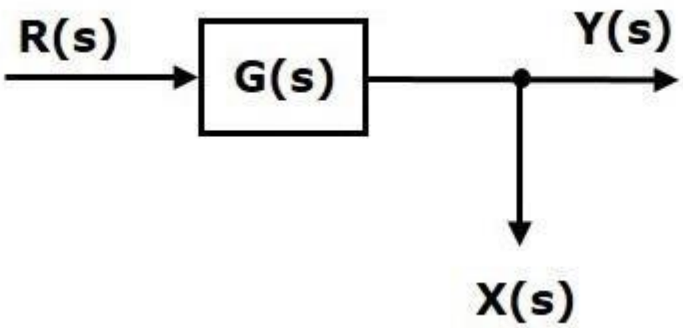
Here, X(s) = R(s) and Y(s) = G(s)R(s)

When you shift the take-off point after the block, the output Y(s) will be same. But, there is difference in X(s) value. So, in order to get the same X(s) value, we require one more block 1/G(s). It is having the input Y(s) and the output is X(s). This block diagram is shown in the following figure.



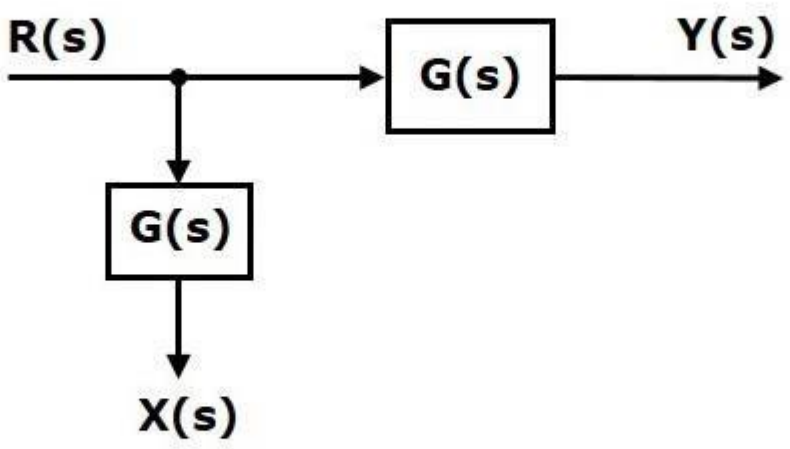
Rule 5b: Shifting Take-off Point Before the Block

Consider the block diagram shown in the following figure. Here, the take-off point is present after the block.



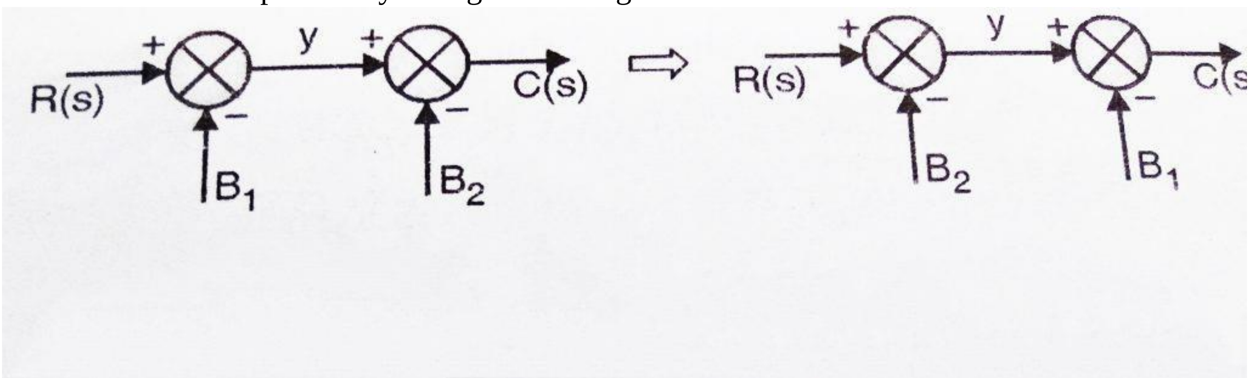
Here, X(s) = Y(s) = G(s)R(s)

When you shift the take-off point before the block, the output Y(s) will be same. But, there is difference in X(s) value. So, in order to get same X(s) value, we require one more block G(s). It is having the input R(s) and the output is X(s). This block diagram is shown in the following figure.



Rule 6: Associative Law For Summing Point

This can be better explained by taking below diagram



Y= R(s) – B1
C(s) = y – B2 = R(s) – B1 – B2
This law is applicable only to summing points which are connected directly to each other. **Note**: If there is a block present between two summing points(and hence they are not connected directly) then this rule can't be applied.

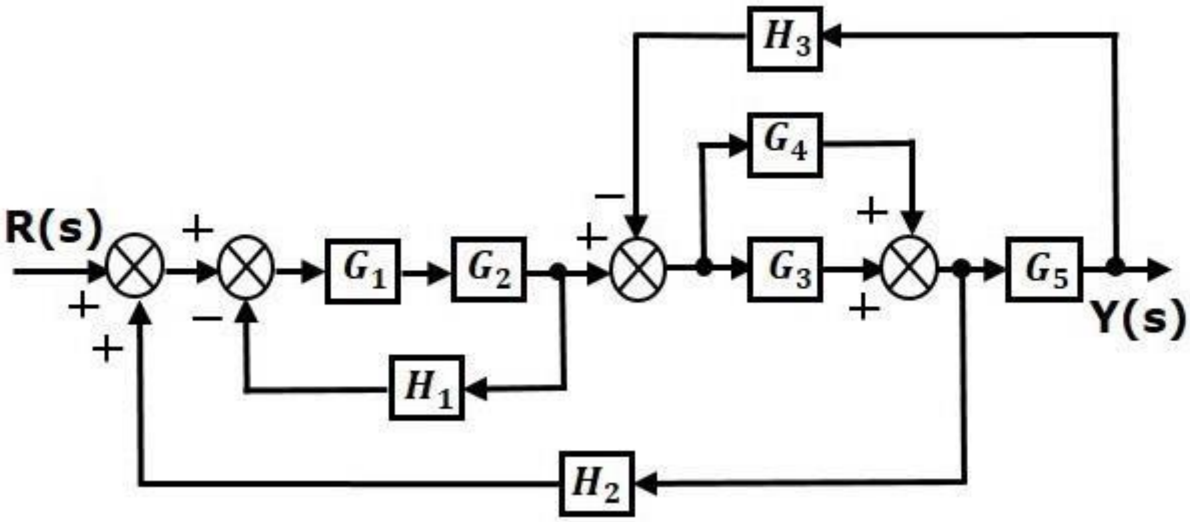
Procedure for finding TF by using Block Diagram Reduction Rules

Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.

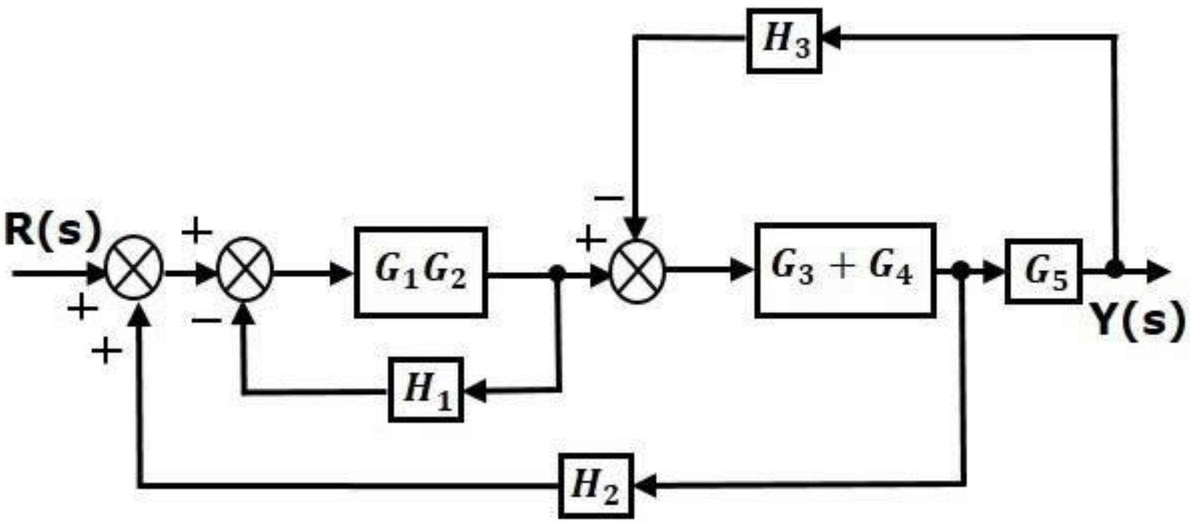
- **Rule 1** − Check for the blocks connected in series and simplify.
- **Rule 2** − Check for the blocks connected in parallel and simplify.
- **Rule 3** − Check for the blocks connected in feedback loop and simplify.
- **Rule 4** − If there is difficulty with take-off point while simplifying, shift it towards right or left of the given block which one is suitable.
- **Rule 5** − If there is difficulty with summing point while simplifying, shift it towards right or left of the given block which one is suitable.
- **Rule 6** − Repeat the above steps till you get the simplified form, i.e., single block.

Example

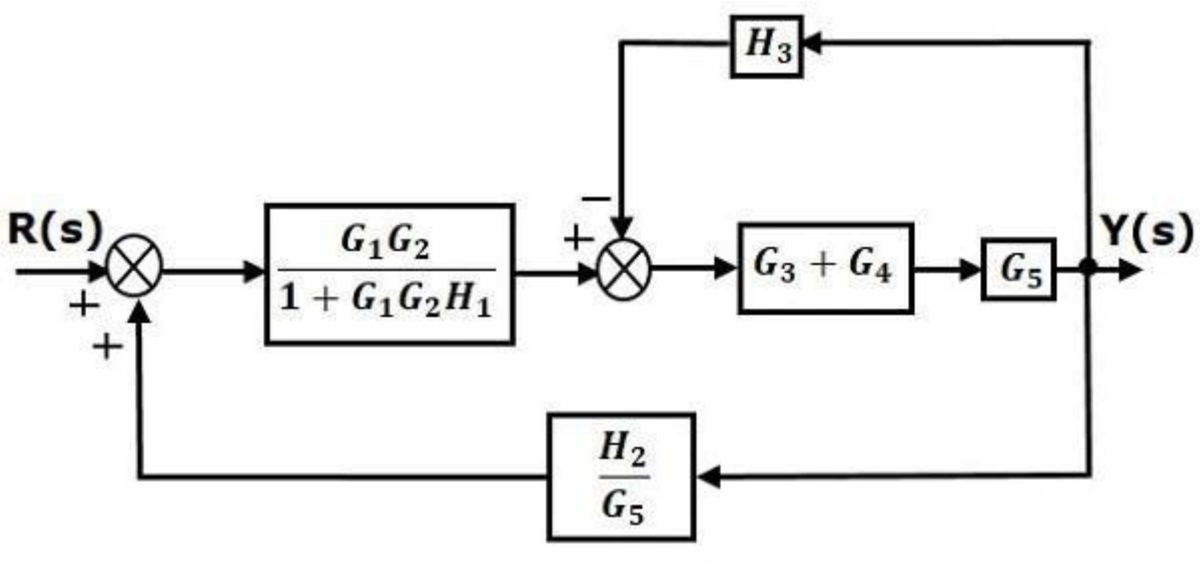
Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



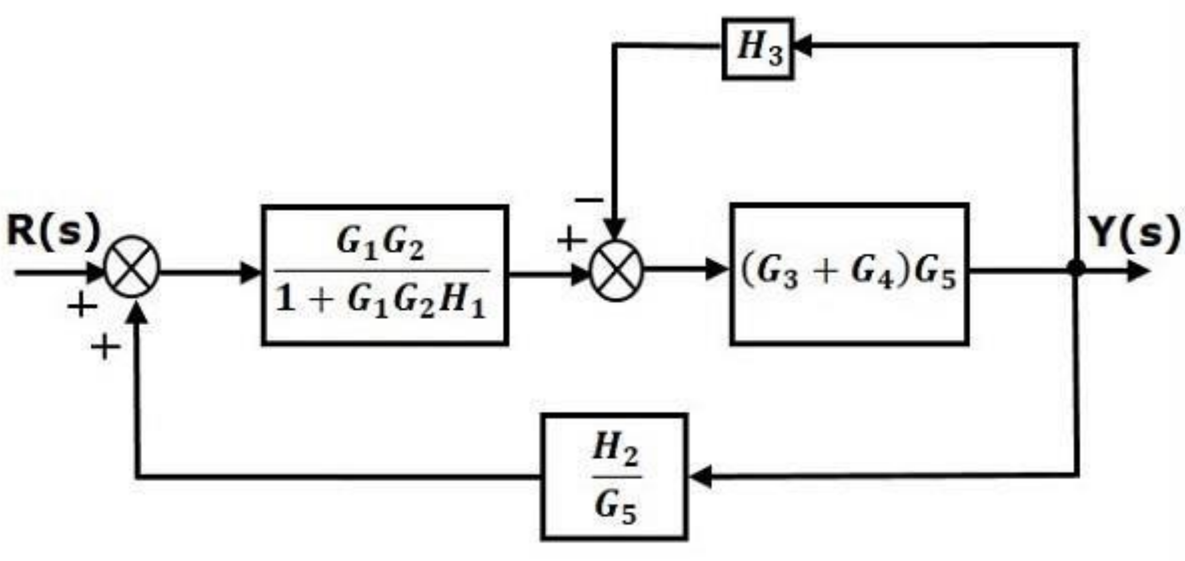
Step 1 − Use Rule 1 for blocks G1 and G2. Use Rule 2 for blocks G3 and G4. The modified block diagram is shown in the following figure.



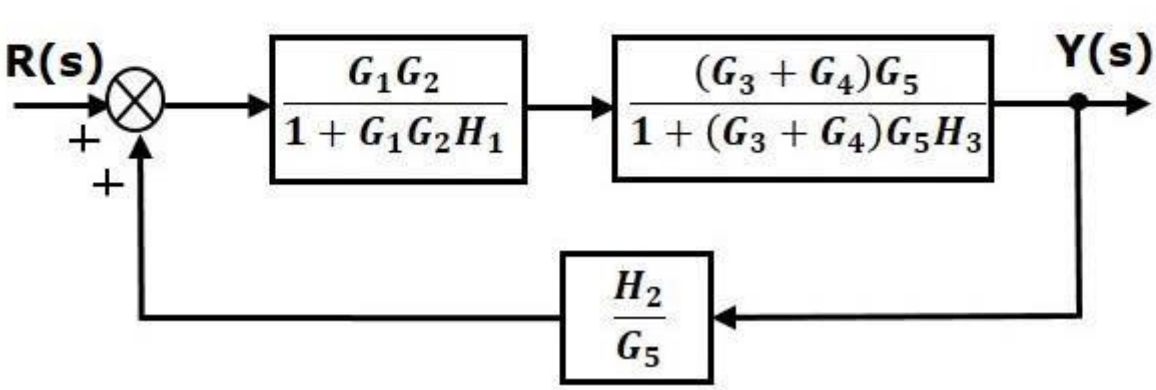
Step 2 − Use Rule 3 for blocks G1G2 and H1. Use Rule 4 for shifting take-off point after the block G5. The modified block diagram is shown in the following figure.



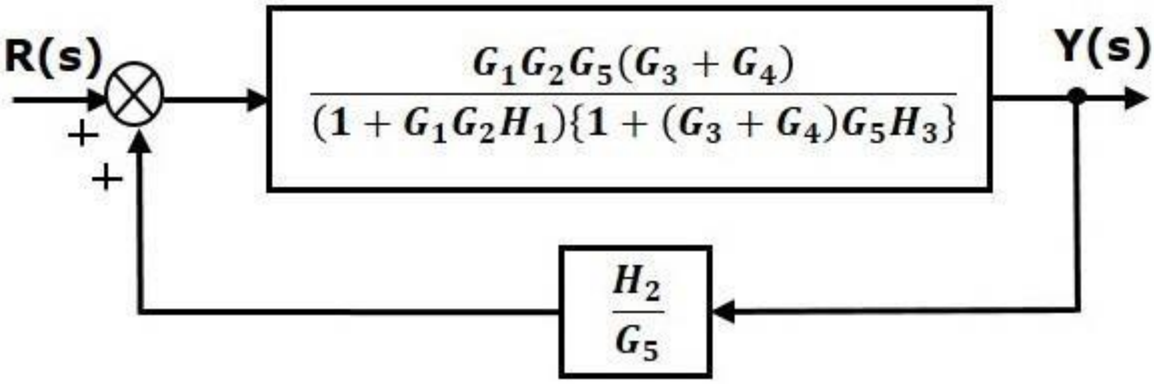
Step 3 − Use Rule 1 for blocks (G3+G4) and G5. The modified block diagram is shown in the following figure.



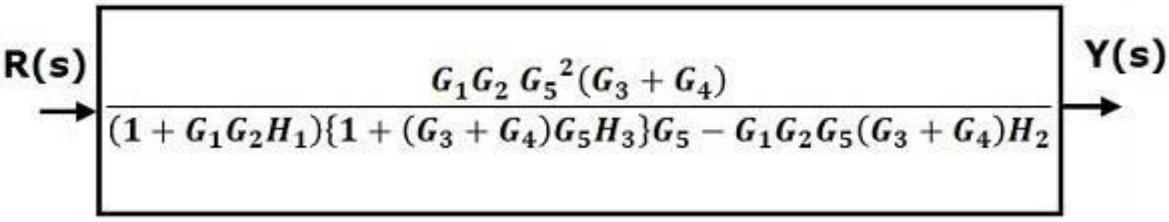
Step 4 − Use Rule 3 for blocks (G3+G4)G5 and H3. The modified block diagram is shown in the following figure.



Step 5 − Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.



Step 6 − Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.



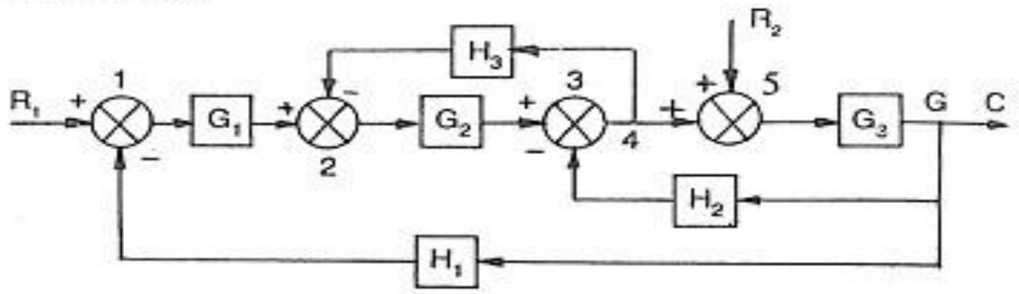
Therefore, the transfer function of the system is

$$Y(s)/R(s)=G_1G_2G_5^2\,(G_3+G_4)/(1+G_1G_2\,H_1)\{1+(G_3+G_4)\,G_5H_3\}G_5-G_1G_2G_5\,(G_3+G_4)H_2$$

Note − Follow these steps in order to calculate the transfer function of the block diagram having multiple inputs.

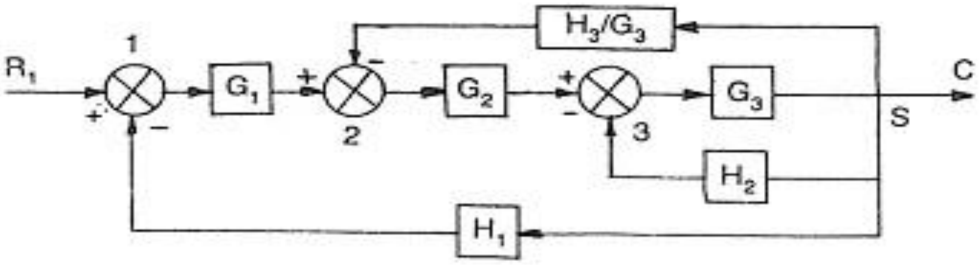
- **Step 1** − Find the transfer function of block diagram by considering one input at a time and make the remaining inputs as zero.
- **Step 2** − Repeat step 1 for remaining inputs.
- **Step 3** − Get the overall transfer function by adding all those transfer functions.

Problem Evaluate $\frac{C}{R_1}$ and $\frac{C}{R_2}$ for a system whose block diagram representation is shown in Fig. R_1 is the input to summing point No. 1.

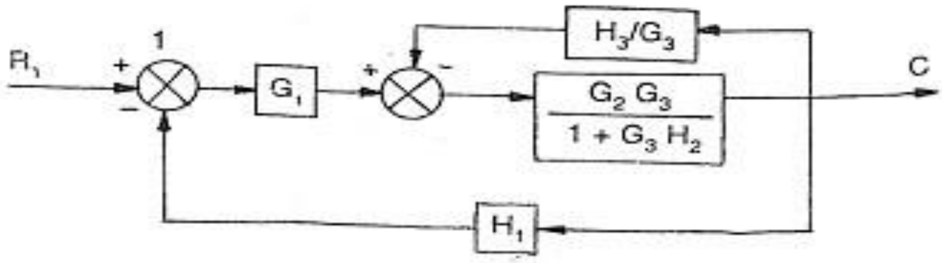


Solution

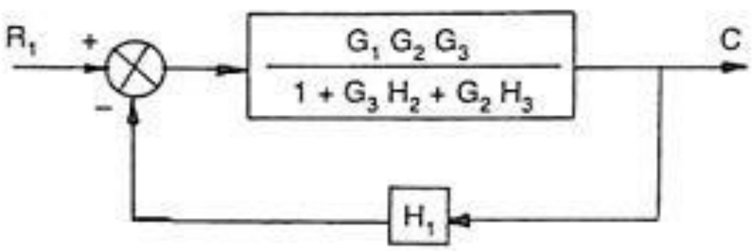
Evaluation of C/R₁ Assume $R_2 = 0$. Therefore summing point No. 5 can be removed. Shift take off point No. 4 beyond block G_3



Eliminate the feedback loop between points **3 and 6**



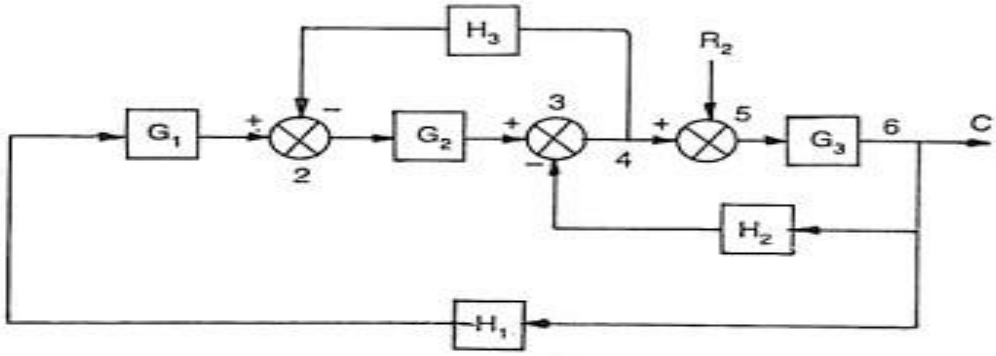
Eliminating the feed back loop again



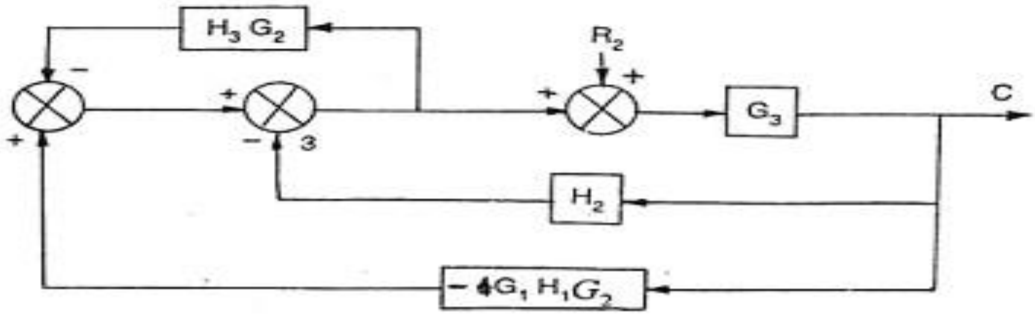
$$\frac{C}{R_1} = \frac{G_1G_2G_3}{1 + G_3H_2 + H_3G_2 + G_1G_2G_3H_1} \quad \textbf{Ans.}$$

Evaluation of C/R₂

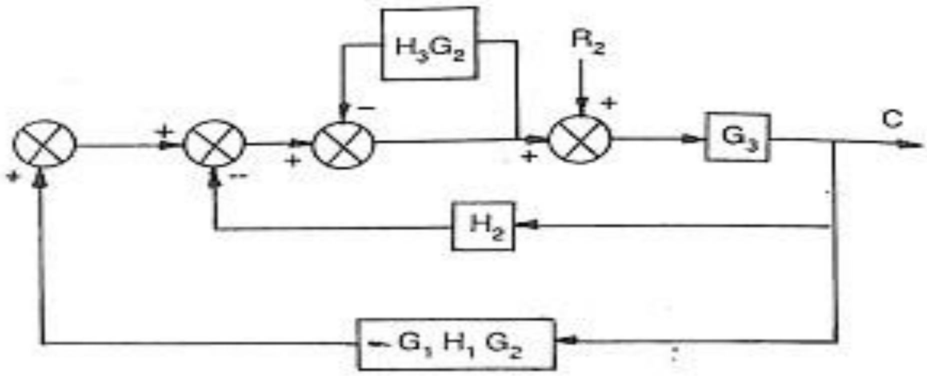
Assume $R_1 = 0$. Thus summing point No. 1 can be removed



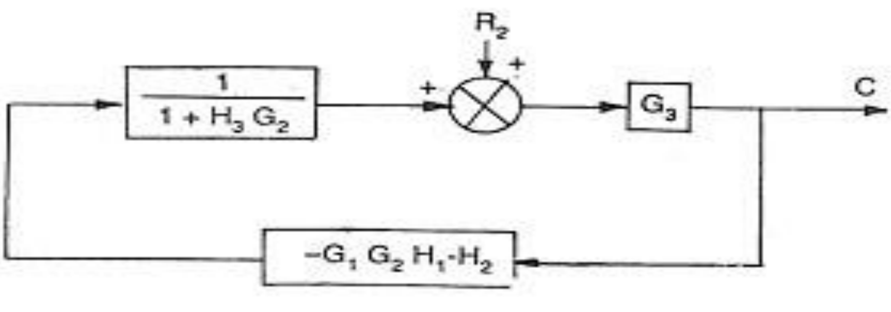
Shifting the summing point No. 2 and rearranging beyond G_2



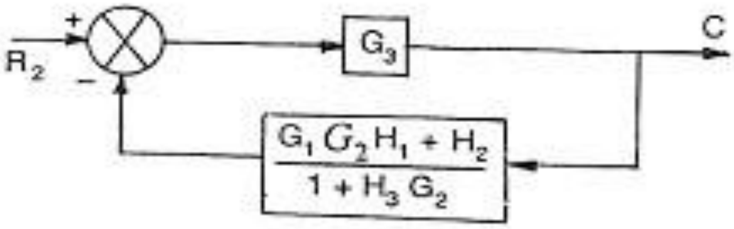
Rearranging, we get



Rearranging and eliminating the feedback loop



Rearranging,



Eliminating the feedback loop, we get

$$\frac{C}{R_2} = \frac{G_3(1 + H_3G_2)}{1 + H_3G_2 + G_3(G_1G_2H_1 + H_2)} \quad \text{Ans.}$$

The block diagram reduction process takes more time for complicated systems. Because, we have to draw the (partially simplified) block diagram after each step. So, to overcome this drawback, use signal flow graphs (representation).

SIGNAL FLOW GRAPHS

Signal flow graph is a graphical representation of algebraic equations. In this chapter, let us discuss the basic concepts related signal flow graph and also learn how to draw signal flow graphs.

Characteristics of SFG: SFG is a graphical representation of the relationship between the variables of a set of linear algebraic equations. It doesn't require any reduction technique or process.

- It represents a network in which nodes are used for the representation of system variable which is connected by direct branches.
- SFG is a diagram which represents a set of equations. It consists of nodes and branches such that each branch of SFG having an arrow which represents the flow of the signal.
- It is only applicable to the linear system.

Terminology used in SFG

Nodes and branches are the basic elements of signal flow graph.

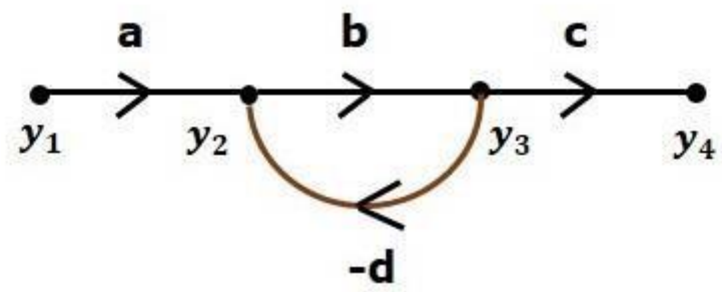
1. Node

Node is a point which represents either a variable or a signal. There are three types of nodes — input node, output node and mixed node.

- **Input Node or source**− It is a node, which has only outgoing branches.
- **Output Node or sink** − It is a node, which has only incoming branches.
- **Mixed Node** − It is a node, which has both incoming and outgoing branches.

Example

Let us consider the following signal flow graph to identify these nodes.



- The **nodes** present in this signal flow graph are **y_1, y_2, y_3** and **y_4**.
- **y_1** and **y_4** are the **input node** and **output node** respectively.
- **y_2** and **y_3** are **mixed nodes**.

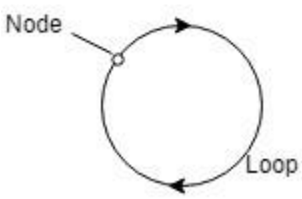
2. Branch

Branch is a line segment which joins two nodes. It has both **gain** and **direction**. For example, there are four branches in the above signal flow graph. These branches have **gains** of **a, b, c** and **-d**.

3. Forward Path

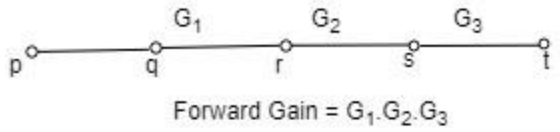
It is a path from an input node to an output node in the direction of branch arrow.

4. Loop: It is a path that starts and ends at the same node.

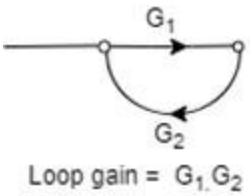


5. Non-touching loop: Loop is said to be non-touching if they do not have any common node.

6. Forward path gain: A product of all branches gain along the forward path is called Forward path gain.



7. Loop Gain: Loop gain is the product of branch gain which travels in the loop.



Construction of Signal Flow Graph

Let us construct a signal flow graph by considering the following algebraic equations −

$y_2 = a_{12}\, y_1 + \; a_{42}\, y_4$

$y_3 = a_{23}\, y_2 + a_{53}\, y_5$

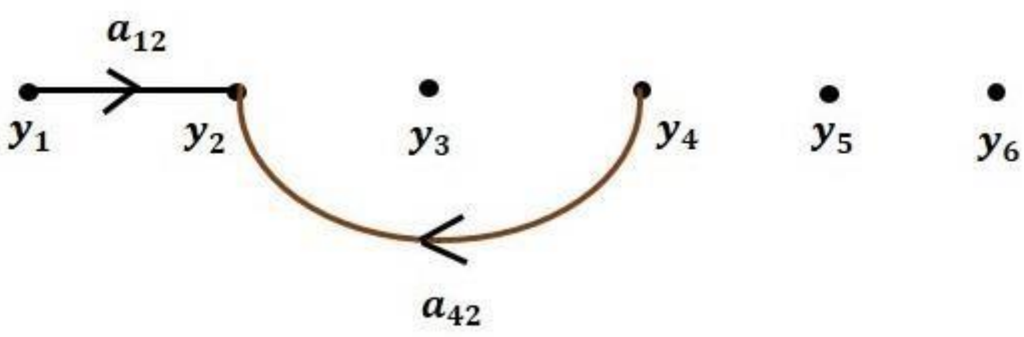
$y_4 = a_{34}\, y_3\; y_5 = a_{45}$

$y_4 + a_{35}\, y_3\; y_6 = a_{56}$

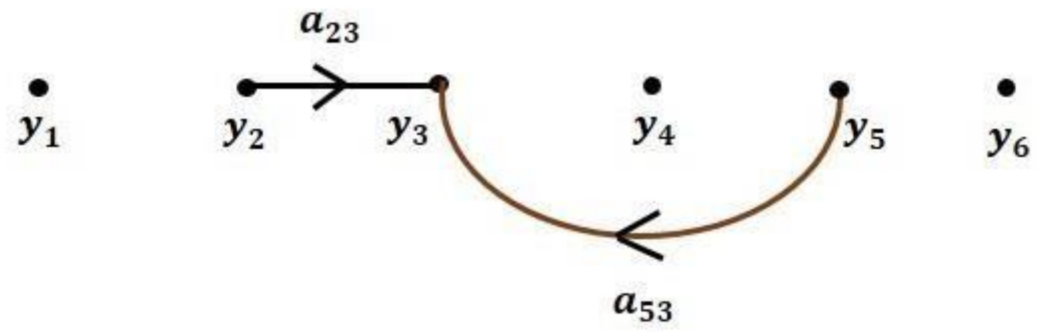
y_5

There will be six **nodes** (y_1, y_2, y_3, y_4, y_5 and y_6) and eight **branches** in this signal flow graph. The gains of the branches are a_{12}, a_{23}, a_{34}, a_{45}, a_{56}, a_{42}, a_{53} and a_{35}.

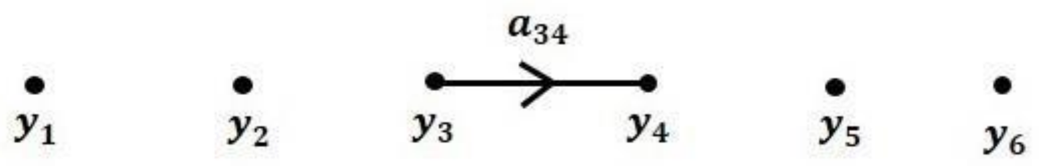
To get the overall signal flow graph, draw the signal flow graph for each equation, then combine all these signal flow graphs and then follow the steps given below − **Step 1** – Signal flow graph for $y_2 = a_{12}\, y_1 + \; a_{42}\, y_4$ is shown in the following figure.



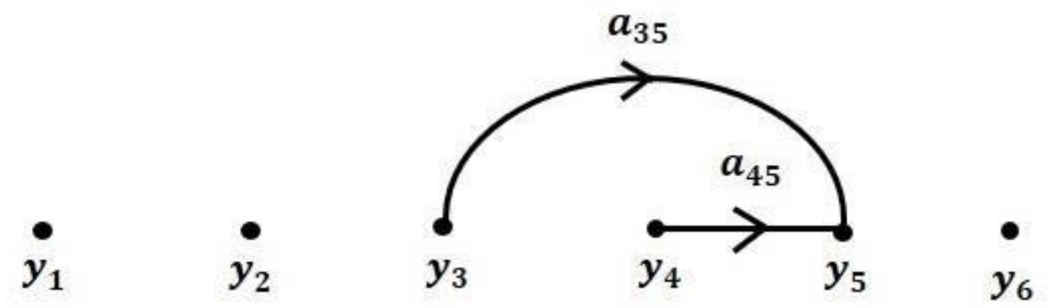
Step 2 – Signal flow graph for $y_3 = a_{23}\, y_2 + a_{53}\, y_5$ is shown in the following figure.



Step 3 − Signal flow graph for $y_4 = a_{34}\ y_3$ is shown in the following figure.

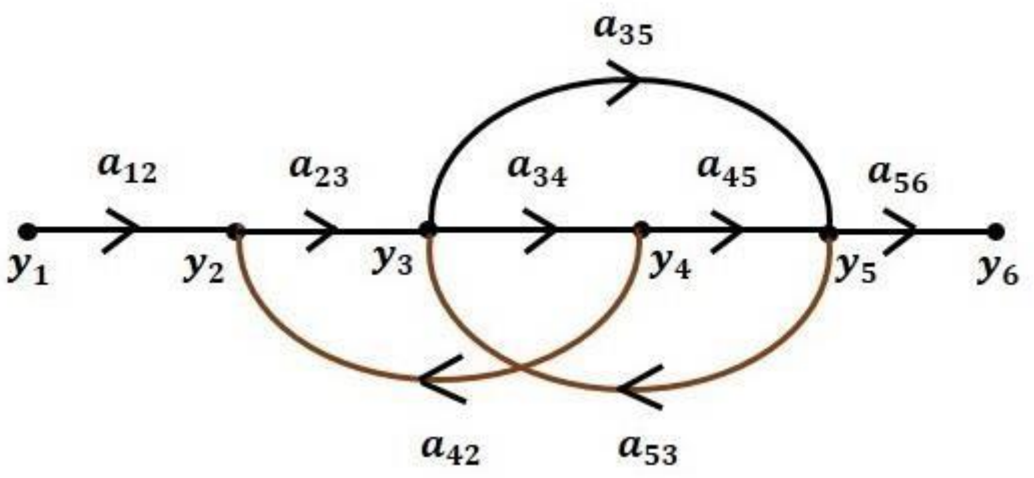


Step 4 − Signal flow graph for $y_5 = a_{45}\ y_4 + a_{35}\ y_3$ is shown in the following figure.



Step 5 − Signal flow graph for $y_6 = a_{56}\ y_5$ is shown in the following figure.

Step 6 − Signal flow graph of overall system is shown in the following figure.



Conversion of Block Diagrams into Signal Flow Graphs

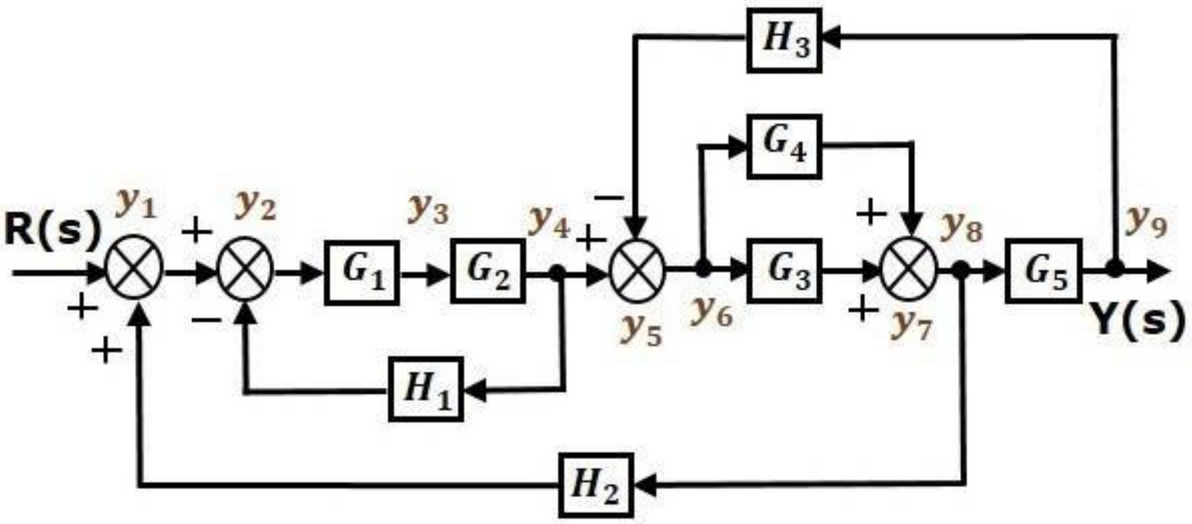
Follow these steps for converting a block diagram into its equivalent signal flow graph.

- Represent all the signals, variables, summing points and take-off points of block diagram as **nodes** in signal flow graph.
- Represent the blocks of block diagram as **branches** in signal flow graph.
- Represent the transfer functions inside the blocks of block diagram as **gains** of the branches in signal flow graph.
- Connect the nodes as per the block diagram. If there is connection between two nodes (but there is no block in between), then represent the gain of the branch as one. **For example**, between summing points, between summing point and takeoff point, between input and summing point, between take-off point and output.

□

Example

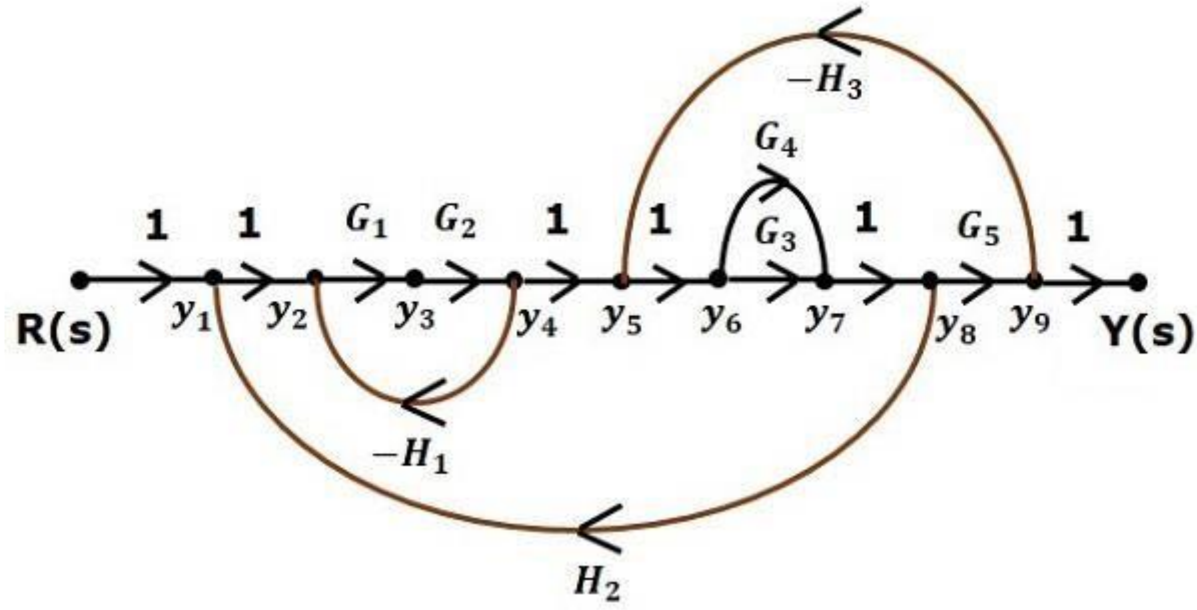
Let us convert the following block diagram into its equivalent signal flow graph.



Represent the input signal R(s) and output signal C(s) of block diagram as input node R(s) and output node C(s) of signal flow graph.

Just for reference, the remaining nodes (y_1 to y_9) are labelled in the block diagram. There are nine nodes other than input and output nodes. That is four nodes for four summing points, four nodes for four take-off points and one node for the variable between blocks G1 and G2.

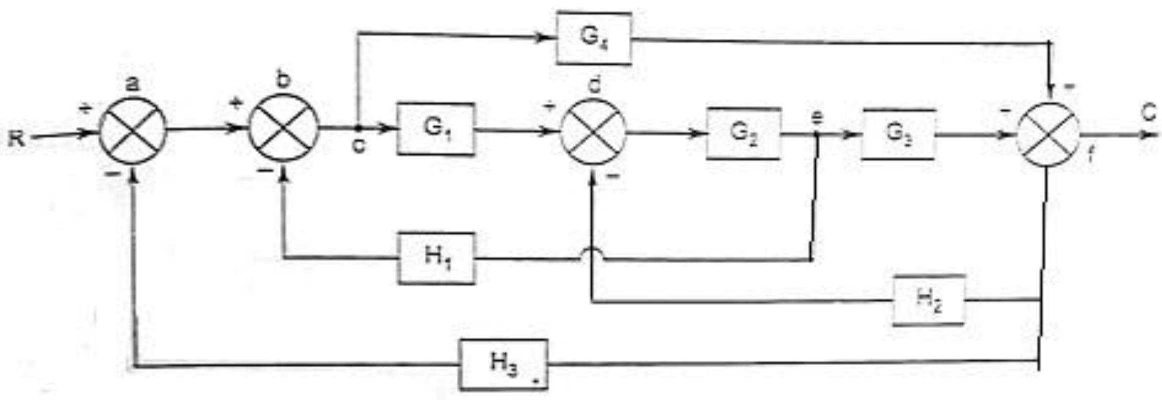
The following figure shows the equivalent signal flow graph.



With the help of Mason's gain formula (discussed in the next chapter), you can calculate the transfer function of this signal flow graph. This is the advantage of signal flow graphs. Here, we no need to simplify (reduce) the signal flow graphs for calculating the transfer function. **Note:** 1. If summing point is present before a take off point it may be assume as same node.

2. If there is a present of summing point in series (no block with in it), it may be take ias same node.

Ex: Determine transfer function by using Mason's gain formula.



Solution:

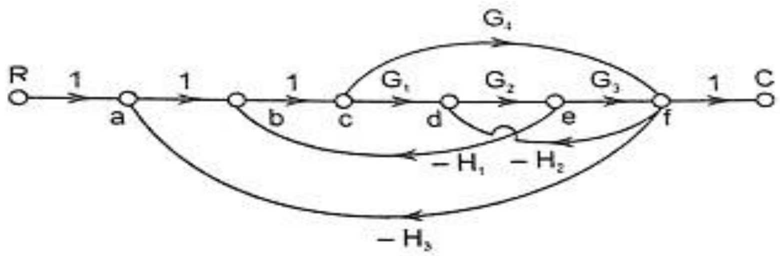


Fig. E3.13(a) *SFG*

Step I : Forward paths of the SFG are as follows :

(*i*) $a-b-c-d-e-f$ $\quad\therefore\quad P_1 = G_1\,G_2\,G_3$

(*ii*) $a-b-c-f$ $\quad\therefore\quad P_2 = G_4$

Step II : Individual loops of the SFG are as follows :

(*i*) $d-e-f-d$ $\quad\therefore\quad L_1 = -\,G_2\,G_3\,H_2$

(*ii*) $b-c-d-e-b$ $\quad\therefore\quad L_2 = -\,G_1\,G_2\,H_1$

(*iii*) $a-b-c-f-a$ $\quad\therefore\quad L_3 = -\,G_4\,H_3$

(*iv*) $b-c-f-d-e-b$ $\quad\therefore\quad L_4 = G_4\,H_2\,G_2\,H_1$

(*v*) $a-b-c-d-e-f-a$ $\quad\therefore\quad L_5 = -\,G_1\,G_2\,G_3\,H_3$

Step III : Gain products of all possible two non-touching loops are as follows :

Loop L_1 and Loop L_2 are non-touching loops

$$\therefore \quad L_{12} = G_2 H_1 G_4 H_2$$

Step IV : $\quad \Delta = 1-(L_1+L_2+L_3+L_4)+L_{12}$

Step V : For $\quad P_1, \Delta_1 = 1$

$$\therefore \frac{C(s)}{R(s)} = \frac{G_1\,G_2\,G_3\,G_4\,G_5}{1-\left(\begin{matrix} G_2\,H_1 + G_4\,H_2 + G_2\,G_3\,G_4\,G_5\,G_6\,G_7\,G_8 \\ + G_2\,G_3\,G_4\,G_5\,G_6\,G_8\,H_4 \end{matrix}\right) + G_1\,G_4\,H_1\,H_2} \quad \textbf{. Ans.}$$

Mason's Gain Formula

Let us now discuss the Mason’s Gain Formula. Suppose there are ‘N’ forward paths in a signal flow graph. The gain between the input and the output nodes of a signal flow graph is nothing but the **transfer function** of the system. It can be calculated by using Mason’s gain formula.

Mason’s gain formula is

$$T = C(s)/R(s) = \left(\frac{1}{\Delta}\right) \sum_{i=1}^{N} P_i \Delta_i$$

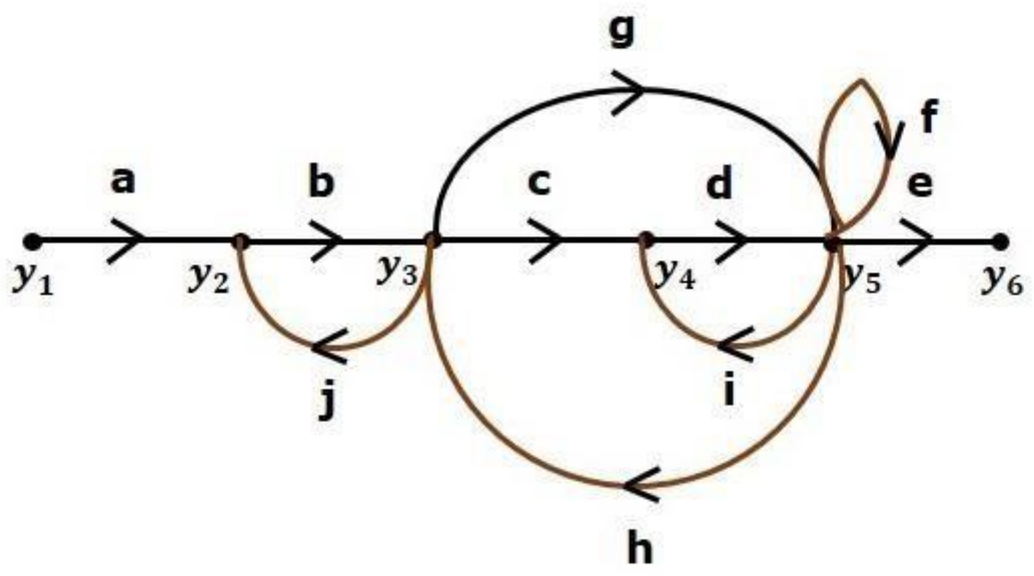
Where,

- **C(s)** is the output node
- **R(s)** is the input node
- **T** is the transfer function or gain between R(s)R(s) and C(s)C(s)
- $\mathbf{P_i}$ is the i^{th} forward path gain

Δ=1−(sum of all individual loop gains) + (sum of gain products of all possible two non touching loops) − (sum of gain products of all possible three non touching loops)+...

Δ_i is obtained from Δ by removing the loops which are touching the i^{th} forward path.

Consider the following signal flow graph in order to understand the basic terminology involved here.



Path

It is a traversal of branches from one node to any other node in the direction of branch arrows. It should not traverse any node more than once.

Examples y2→y3→y4→y5 and y5→y3→y2

Forward Path

The path that exists from the input node to the output node is known as **forward path**.

Examples − y1→y2→y3→y4→y5→y6 and y1→y2→y3→y5→y6.

Forward Path Gain

It is obtained by calculating the product of all branch gains of the forward path.

Examples – abcde is the forward path gain of y1→y2→y3→y4→y5→y6 and abge is the forward path gain of y1→y2→y3→y5→y6.

Loop

The path that starts from one node and ends at the same node is known as **loop**. Hence, it is a closed path.

Examples y2→y3→y2 and y3→y5→y3.

Loop Gain

It is obtained by calculating the product of all branch gains of a loop.

Examples − bj is the loop gain of y2→y3→y2 and gh is the loop gain of y3→y5→y3.

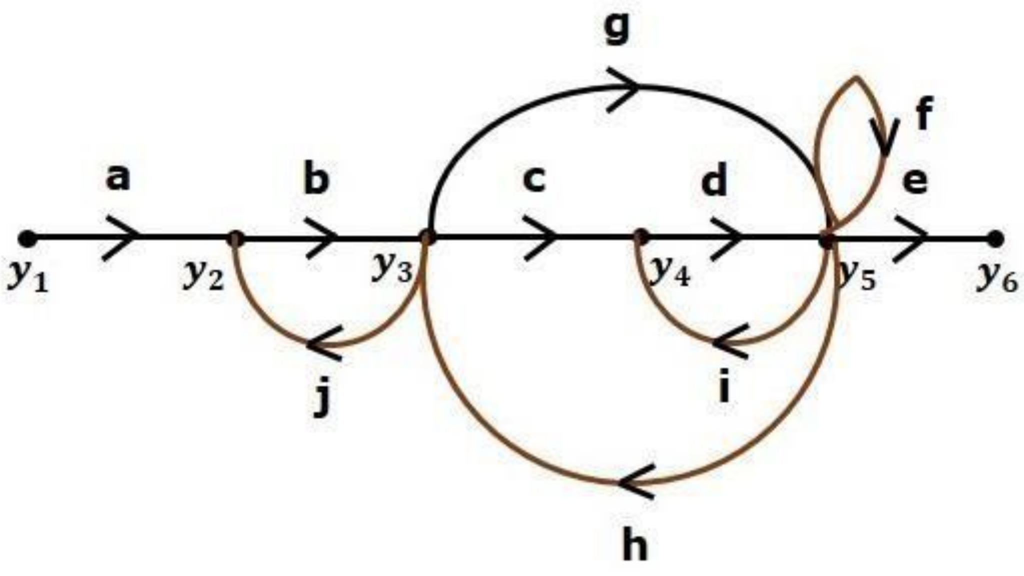
Non-touching Loops

These are the loops, which should not have any common node.

Examples − The loops y2→y3→y2 and y4→y5→y4 are non-touching.

Calculation of Transfer Function using Mason's Gain Formula

Let us consider the same signal flow graph for finding transfer function.



Number of forward paths, N = 2.

- First forward path is - y1→y2→y3→y4→y5→y6. ▯ First forward path gain, p1=abcde.
- Second forward path is - y1→y2→y3→y5→y6. ▯ Second forward path gain, p2=abge.

Number of individual loops, L = 5.

Loops are - y2→y3→y2, y3→y5→y3, y3→y4→y5→y3 , y4→y5→y4 and y5→y5. Loop gains are - l1=bj, l2=gh, l3=cdh, l4=di and l5=f Number

of two non-touching loops = 2.

- First non-touching loops pair is - y2→y3→y2, y4→y5→y4.
- Gain product of first non-touching loops pair, l1l4= bjdi ▯ Second non-touching loops pair is - y2→y3→y2, y5→y5
- Gain product of second non-touching loops pair is - l1l5=bjf

Higher number of (more than two) non-touching loops are not present in this signal flow graph.

We know,

Δ=1− (sum of all individual loop gains)
+ (sum of gain products of all possible two non touching loops)
−(sum of gain products of all possible three non touching loops)+...

Substitute the values in the above equation,

$\Delta = 1 - (bj+gh+cdh+di+f) + (bjdi+bjf) - (0)$
$\Rightarrow \Delta = 1 - (bj+gh+cdh+di+f) + bjdi+bjf$

There is no loop which is non-touching to the first forward path.

So, $\Delta 1 = 1$
Similarly, $\Delta 2 = 1$. Since, no loop which is non-touching to the second forward path.

Substitute, N = 2 in Mason's gain formula

$$T=C(s)R(s)=[P1\Delta 1+P2\Delta 2]/\Delta$$

Substitute all the necessary values in the above equation.

$$T=C(s)R(s)=(abcde)1+(abge)1/[1-(bj+gh+cdh+di+f)+bjdi+bjf]$$

$$\Rightarrow T=C(s)R(s)=(abcde)+(abge)/[1-(bj+gh+cdh+di+f)+bjdi+bjf]$$

Therefore, the transfer function is -

$$T=C(s)R(s)=(abcde)+(abge)/[1-(bj+gh+cdh+di+f)+bjdi+bjf]$$