

## QUESTION BANK

### MATRICES AND DETERMINANT

#### SHORT ANSWER TYPE QUESTION (2MARKS AND 5MARKS)

1. Solve  $\begin{vmatrix} 4 & x+1 \\ 3 & x \end{vmatrix} = 5$ .

2. Find the value of  $\begin{vmatrix} -6 & 0 & 0 \\ 3 & -5 & 7 \\ 2 & 8 & 11 \end{vmatrix}$ .

3. Find the minimum value of  $\begin{vmatrix} \sin x & \cos x \\ -\cos x & 1 + \sin x \end{vmatrix}$ .

4. If  $\begin{vmatrix} a & b & c \\ b & a & b \\ x & b & c \end{vmatrix} = 0$ , find x.

5. Solve by Cramer's rule  $2x - y = 3$ ,  $x + 2y = 4$ .

Find the value of  $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ .

6. Construct a  $2 \times 3$  matrix having elements given by  $a_{ij} = i + j$ .

7. Find x and y when  $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ .

8. Find the adjoint of matrix  $\begin{bmatrix} 1 & -w \\ w^2 & 1 \end{bmatrix}$ .

9. Given  $[x \ y \ z] - [-4 \ 3 \ 1] = [-5 \ 1 \ 0]$ .

10. Write down the matrix  $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$  if  $a_{ij} = 2i + 3j$ .

11. Find the value of the determinant  $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ .

12. Solve the determinant  $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$ .

13. Prove that  $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ .

14. Solve by Cramer's rule  $x+y+z=3$

$$2x+3y+4z=9$$

$$x+2y-4z=-1.$$

15. Prove that  $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$ .

16. Find the inverse of the matrix  $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ .

17. Find the adjoint of the matrix  $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 1 & 3 & -2 \end{bmatrix}$

18. Find the inverse of the matrix  $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ .

19. Solve by matrix method  $x-y+z=4$ ,  $2x+y-3z=0$ ,  $x+y+z=2$ .

### LONG QUESTIONS (10 MARKS)

1. (a) find the adjoint of the matrix  $\begin{bmatrix} -2 & 2 & 3 \\ 1 & 4 & 2 \\ -2 & -3 & 1 \end{bmatrix}$ .

(b) Solve by matrix method  $x+y-z=6$ ,  $2x-3y+z=1$ ,  $2x-4y+2z=1$ .

2. Prove that  $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$ .

3. Solve by Cramer's rule  $x-y+z=1$ ,  $2x+3y-5z=7$ ,  $3x-4y-2z=-1$ .

4. Prove that  $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$ .

5. (a) Verify  $(AB)^T = B^T A^T$  where  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$ .

(b) Find the inverse of  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ .

### TRIGONOMETRY

#### SHORT ANSWER TYPE QUESTION (2MARKS AND 5MARKS)

1. Find the value of  $\tan(-840)^\circ$ .
2. Find the value of  $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$ .
3. Prove that  $2\sin 105^\circ \cdot \sin 15^\circ$ .
4. Find the value of  $\frac{\tan 15^\circ}{1 - \tan^2 15^\circ}$ .
5. If  $\tan \alpha = \frac{1}{2}$ ,  $\tan \beta = \frac{1}{3}$  then find the value of  $(\alpha + \beta)$ .
6. If  $\frac{1 + \sin A}{\cos A} = \sqrt{2} + 1$ , then find the value of  $\frac{1 - \sin A}{\cos A}$ .
7. Find the minimum value of  $\sin \theta \cdot \cos \theta$ .
8. Find the value of  $\tan \frac{\pi}{20} \tan \frac{3\pi}{20} \tan \frac{5\pi}{20} \tan \frac{7\pi}{20} \tan \frac{9\pi}{20}$ .
9. Find the value of  $\sin^2 24^\circ - \sin^2 6^\circ$ .
10. Find the value of  $\cos \left[ \sin^{-1} \left( \frac{-1}{2} \right) \right]$ .
11. Prove that  $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = \frac{3}{16}$ .
12. Find the value of  $\sin 18^\circ$  and  $\cos 36^\circ$ .
13. If  $A+B+C=\pi$ , then prove that  $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \cdot \sin B \cdot \sin C$
14. If  $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$  show that  $xy + yz + zx = 1$ .
15. Prove that  $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \tan 54^\circ$ .
16. Find the maximum and minimum values of  $5\sin x + 12 \cos x$ .
17. Find the value of  $\tan 75^\circ$  and hence prove that  $\tan 75^\circ + \cot 75^\circ = 4$
18. Prove that  $\sin^{-1} \frac{4}{5} + 2 \tan^{-1} \frac{1}{3} = \frac{\pi}{2}$ .

#### LONG QUESTIONS (10 MARKS)

1. If  $A+B=45^\circ$  Prove that

- (i)  $(1+\tan A)(1+\tan B)=2$ . Deduce the value of  $\tan 22\frac{1}{2}^\circ$  .
- (ii)  $(\cot A - 1)(\cot B - 1)=2$ .
2. (a) Prove that  $\cot 7\frac{1}{2}^\circ = \sqrt{6} + \sqrt{3} + \sqrt{2} + 2$  .
- (b) Prove that  $\sin A + \sin B + \sin C = 4 \sin \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$  .
3. If  $\sin A = K \sin B$ , prove that  $\tan \frac{1}{2}(A - B) = \frac{K-1}{K+1} \tan \frac{1}{2}(A + B)$ .
4. Prove that  $2 \cos \frac{\pi}{16} = \sqrt{2 + \sqrt{2 + \sqrt{2}}}$  .
5. (i) If  $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$ , show that  $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2}$ .
- (ii) If  $\sec(\theta + \alpha) + \sec(\theta - \alpha) = 2 \sec \theta$ , show that  $\cos \theta = \sqrt{2} \cos \frac{\alpha}{2}$  .
6. (a) Prove that  $\cos^2 A + \cos^2 B + 2 \cos A \cdot \cos B \cdot \cos C = \sin^2 C$  .
- (b) Prove that  $2 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{4} = \tan^{-1} \frac{8}{53}$  .

## TWO DIMENSIONAL GEOMETRY

### SHORT ANSWER TYPE QUESTION (2MARKS AND 5MARKS)

1. Find the distance between the points P(-3,2) and Q(2,-1) .
2. If the area of the triangle with the vertices (0,0), (1,0), (0,a) is 10 units, find the value of a?
3. Find the equation of a line which cuts off an intercept -2 on the axis of "y" and makes an angle  $45^\circ$  with positive direction of x-axis.
4. Find the co-ordinate of the point dividing the joining of (3,7) and (-1,-5) internally in the ratio 2:3.
5. Find the equation of the line passing through (-1,2) and making intercepts on the y-axis.
6. Reduce  $3x+5y+4=0$  to the intercept form and y-intercept.
7. Find the centre and radius of the circle  $2x^2 + 2y^2 - 5x + 3y - 11 = 0$ .
8. Determine the distance between the parallel lines  $x+5=0$  and  $x-5=0$ .
9. Find the equation of a circle with centre (-3, 2) and radius 7.
10. Determine the equation of the straight line parallel to x-axis and passing through (3,4).

11. Find the equation of a straight line that passes through the point (3,4) and perpendicular to the line  $3x+2y+5=0$ .
12. Find the equation of line passing through the point of intersection of lines  $x+3y+2=0$  and  $x-2y-4=0$  and perpendicular of the line  $2y+5x-9=0$ .
13. Find the equation of straight line passing through (-2,3) and sum whose intercept is 2.
14. Find the equation of bisecting the line segment joining (3,-4) and (1,2) at right angle.
15. Show that the points A(-1,4),B(0,2),C(2,-2) are collinear.
16. Find the equation of a circle whose end points of diameter are (-5, 3) and (7,5).
17. Find the equation of the line through the point of intersection of  $3x+4y-7=0$  and  $x-y+2=0$  and which is parallel to the line  $5x-y+11=0$ .
18. Show that the points (1,1),(4,4),(4,8) and (1,5) are the vertices of the parallelogram.
19. Find the co-ordinate of the point which divide internally and externally the line joining (1,-3) and (-3,9) in the ratio 1:3.
20. Find the equation of the circle passing through the points (3, 4) (4, -3) and (-3, 4).

**LONG QUESTIONS (10 MARKS)**

- 1.(a) Find the equation of the line passing through the intersection of  $2x - y - 1 = 0$  and  $3x-4y+ 6 = 0$  and parallel to the line  $x + y - 2 = 0$   
 (b) Find the equation of the circle passing through the points (1,-2) and its centre at the point of intersection of lines  $2x-y+3=0$  and  $x+2y-1=0$  .
2. (a) Find the co-ordinates of the foot of the perpendicular from the point (2, 3) on the line  $3x-4y+7=0$   
 (b) Find the equation of the line passing through (-4, 2) and parallel to the line  $4x-3y=0$ .
3. Find the equations of straight lines passing through the point (3,-2) and making an angle  $45^\circ$  with the line  $6x+5y=1$ .
4. Find the distance of the point (3,2) from the line  $x+y-1=0$ , measured parallel to the line  $3x-4y+1=0$  .
5. Find the equation of the circle whose Centre is on the line  $8x+5y=0$  and the circle passing through the points (2,1) and (3,5) .

### **THREE DIMENSIONAL GEOMETRY**

1. Determine the Centre and radius of the sphere  $x^2 + y^2 + z^2 - 4x + 6y - 8z + 1 = 0$ .
2. Determine the value of  $k$  such that the planes  $x + 3y + kz = 5$  and  $kx + y + 2z = 0$  are perpendicular to each other.
3. Find the image of the point  $(-6, 2, -3)$  w.r.t  $yz$ -plane.
4. Find the value of  $k$  such that the points  $(1, -2, 3)$ ,  $(3, -1, 2)$  and  $(7, 1, k)$  are collinear .
5. Find out the equation of the plane passing through  $(1, 1, 2)$  and parallel to  $x + y + z - 1 = 0$ .
6. Find the distance between the parallel planes  $x - y + z + 1 = 0$  and  $-z - x + 1 = 0$  .
7. Find the equation of the sphere with Centre  $(3, -2, 5)$  and radius 4.
8. Find the direction cosines of the line passing through the two points  $(-2, 4, -5)$  and  $(1, 2, 3)$ .
9. Find the distance of the point  $P(x, y, z)$  from  $z$ -axis.
10. Find the projection of the line segment joining  $(1, 3, -1)$  and  $(3, 2, 4)$  on  $z$ -axis .
11. Find the equation of the plane which passes through the point  $(1, -1, 4)$  and is parallel to the Plane  $2x + 3y + 7z = 11$ .
12. Find the angle between two planes  $2x + 2y - 3z = 5$  and  $3x - 3y + 5z = 3$ .
13. Find the foot of the perpendicular drawn from the point  $(0, 0, 0)$  on the plane  $2x + y + z - 3 = 0$ .
14. Find the equation of the sphere on the join of  $(2, 3, 5)$  and  $(4, 9, -3)$  as diameter ?
15. Find the equation of the sphere with its centre at  $(1, -2, 3)$  and touching the plane  $2x - 3y + z + 6 = 0$ .
16. Show that points  $(0, 1, 2)$ ,  $(2, 5, 8)$ ,  $(5, 6, 6)$  and  $(3, 2, 0)$  are the vertices of the parallelogram.
17. Find the ratio in which the line joining the points  $(2, -3, 1)$ ,  $(3, -4, -5)$  is bisected by the plane  $2x + y + z = 7$ .
18. Show that  $A(0, 0, 0)$ ,  $B(3, 4, 5)$ ,  $C(-3, -4, -5)$  are collinear.

### **LONG QUESTIONS (10 MARKS)**

1. Find the equation of the sphere passing through the point  $(1, 2, -3)$  and  $(3, -1, 2)$  and centre lying on  $y$ -axis.
2. Show that the points  $A(1,2,3), B(-1,-2,-1), C(2,3,2)$  and  $D(4,7,6)$  are the vertices of a parallelogram  $ABCD$ , but it is not a rectangle.
3. Find the equation of the plane which is perpendicular to the plane  $5x+3y+6z+8=0$  and contains the line of intersection of the plane  $x+2y+3z-4=0$  and  $2x+y-z+5=0$ .
4. Find the equation of the sphere which passes through the points  $(0,0,0), (0,1,0), (1,0,0)$  and  $(0,0,1)$ .
5. (a) Find the equation of sphere with its centre at  $(1,-2,3)$  and touching the plane  $2x-3y+z+6=0$ .  
(b) Find the equation of the plane through the points  $(2,1,0)$  and passing through intersection of the planes  $3x-2y+z-1=0$  and  $x-2y+3z-1=0$
6. Find the equation of the plane containing the line of intersection of the plane  $x+y+z+1=0$ ,  $2x-3y+5z-2=0$  and passing through the point  $(-1,2,1)$ .