

SUB-MATHEMATICS

2ND SEMESTER

CHAPTER - LIMIT & CONTINUITY

2 Marks

Q.1 Evaluate $\lim_{n \rightarrow \infty} \frac{n!}{(n+1)! - n!}$

Q.11 Evaluate $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right)$

Q.2 Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$

Q.12 Evaluate $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$

Q.3 Evaluate $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 3x}$

Q.13 Evaluate $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1} - 1}$

Q.4 Evaluate $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$

Q.14 Prove that $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

Q.5 Evaluate $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$

Q.15 Evaluate the L.H.L of the function

Q.6 Evaluate $\lim_{x \rightarrow 5} \frac{\sqrt{x} - \sqrt{5}}{x - 5}$

$f(x) = \begin{cases} \frac{|x-4|}{x-4}, & x \neq 4 \\ 0, & x = 4 \end{cases}$ at $x = 4$

Q.7 Find $\lim_{x \rightarrow \sqrt{2}} [x]$

Q.8 Evaluate $\lim_{x \rightarrow 0} \frac{x}{\tan^{-1} x}$

Q.9 Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$

Q.10 Evaluate $\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x}$

5 Marks

Q.1 Evaluate $\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^4}$

Q.2 Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sin^{-1} x}$

Q.3 $\lim_{x \rightarrow 0} \frac{(x+9)^{3/2} - 27}{x}$

Q.4 $\lim_{x \rightarrow 0} \frac{\operatorname{cosec} x - \cot x}{x}$

Q.5 Evaluate $\lim_{x \rightarrow 1} \frac{\log_e (2x-1)}{x-1}$

Q.6 Examine the continuity of the function 'f' defined by

$$f(x) = \begin{cases} 2x+1, & \text{if } x < 1 \\ 0, & \text{if } x = 0 \\ x^2-1, & \text{if } x > 1 \end{cases}$$

Q.7 Discuss the continuity of the function.

$$f(x) = \begin{cases} x - \frac{|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases} \quad \text{at } x=0$$

Q.8 Examine the continuity of the function.

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases} \quad \text{at } x=0$$

Q.9 If a function $f(x)$ is defined as $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$

is continuous at $x=3$

then find the value of k .

Q.10 Evaluate $\lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{\sin^3 x} \right)$

10 Marks

Q.1 Examine the continuity of the function

$$f(x) = \begin{cases} \frac{e^{2x} - 1}{e^{2x} + 1}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases} \quad \text{at } x=0$$

Q.2 If $f(x) = \begin{cases} ax^2 + b, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ 2ax - b, & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$,

then find a & b .

Q.3 Discuss the continuity of the function at $x = \frac{1}{2}$,

$$f(x) = \begin{cases} x, & 0 \leq x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x \leq 1 \end{cases}$$

Q.4 Find the value of a if $\lim_{x \rightarrow 2} \frac{\log(2x-3)}{a(x-2)} = 1$

CHAPTER - DERIVATIVE

2 marks Find the derivative of the followings.

(i) $y = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$

(ii) $y = \log(\log(\log x))$

(iii) $y = \sqrt{e^{\sqrt{x}}}$

(iv) $y = \sqrt{\cot^{-1} \sqrt{x}}$

(v) $y = \frac{a^x - b^x}{x}$

(vi) Find y_1 and y_2 if $y = \tan x$.

(vii) If $y = A \cos x + B \sin x$ then P.T. $\frac{d^2 y}{dx^2} + y = 0$

(viii) If $z = x^2 y + xy^2$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

(ix) State Euler's Theorem.

(x) Test whether the function $f(x, y) = \sin^{-1}\left(\frac{x}{y}\right)$ is homogeneous or not.

(xi) Find the partial derivatives of $u = e^{xyz}$ & $u = x^y + y^x$

(xii) What is the slope of the curve $y = \sin x$ at $x = \frac{\pi}{6}$.

(xiii) Differentiate $\sin[\cos(\tan x)]$

(xiv) If $f(x) = \sin^2 x$, find $f'(x)$.

(xv) Find the derivative of $\sin x^\circ$.

5 Marks

Q.1 Differentiate $\tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$

Q.2 Differentiate $\tan^{-1}(\sqrt{1+x^2} + x)$

Q.3 Differentiate $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

Q.4 Find $\frac{dy}{dx}$ if $x = a(\theta + \sin \theta)$, $y = a \cos \theta$

Q.5 Find $\frac{dy}{dx}$, if $e^y \ln x + x \ln y = 0$

Q.6 Differentiate $\sin^2 x$ w.r.t. $(\ln x)^2$.

Q.7 Find $\frac{dy}{dx}$ if $x^y y^x = 1$

Q.8 Differentiate (i) $(\ln x)^x$ (ii) $(\ln x)^{\tan x}$ (iii) $(\sin x)^{\cos x}$.

Q.9 If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, find $\frac{d^2 y}{dx^2}$.

Q.10 If $y = \log(x + \sqrt{1+x^2})$, P.T. $(1+x^2)^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0$

Q.11 If $y = \frac{1}{2}(\sin^{-1} x)^2$, then P.T. $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = 1$

Q.12 If $x = f\left(\frac{y}{x}\right)$, show that $x \frac{\partial x}{\partial x} + y \frac{\partial x}{\partial y} = 0$

Q.13 Differentiate $\tan^{-1}(\sec x + \tan x)$

Q.14 Prove that if $z = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$ then $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \tan z$

Q.15 If $f(x, y, z) = \log(x^3 + y^3 + z^3 - 3xyz)$ P.T.

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 3$$

10 Marks

Q.1 If $y = e^{m \cos^{-1} x}$, then show that $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$

Q.2 If $x = \sec t$, $y = \sin(pt)$ then show that

$$(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$$

Q.3 If $\sqrt{1-x^4} + \sqrt{1-y^4} = k(x^2 - y^2)$, then show that $\frac{dy}{dx} = \frac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$

Q.4 Differentiate $(\cos x)^{\ln x} + (\log x)^x$

Q.5 Differentiate $(\cos x)^y = (\sec y)^x$

Q.6 (i) Differentiate $\frac{e^{x^2} \tan^{-1} x}{\sqrt{1+x^2}}$

(ii) find $\frac{dy}{dx}$, when $x^y = y^x$

Q.7 If $u = (x^2 + y^2 + z^2)^{-1/2}$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

Chapter - Integration

2 Marks

- ① Evaluate $\int \sqrt{1 - \sin 2x} \, dx$.
- ② Evaluate $\int \sin^{-1}(\cos x) \, dx$.
- ③ Evaluate $\int (\tan x + \cot x)^2 \, dx$.
- ④ Evaluate $\int \frac{x^2}{x^2 + 1} \, dx$.
- ⑤ Evaluate $\int \frac{\operatorname{cosec}^2 x}{1 + \cot x} \, dx$.
- ⑥ Evaluate $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} \, dx$.
- ⑦ Evaluate $\int 2x \operatorname{cosec}^2 x^2 \, dx$.
- ⑧ Evaluate $\int \frac{\operatorname{cosec}^2(\ln x)}{x} \, dx$.
- ⑨ Evaluate $\int \frac{dx}{x \sqrt{25 - (\ln x)^2}}$.
- ⑩ Evaluate $\int \frac{dx}{x \ln x \sqrt{(\ln x)^2 - 4}}$.
- ⑪ Evaluate $\int \frac{e^{5x}}{\sqrt{e^{10x} - 4}} \, dx$.
- ⑫ Evaluate $\int_0^1 \frac{dx}{1+x^2}$.
- ⑬ Evaluate $\int_2^3 2x e^{x^2} \, dx$.
- ⑭ Evaluate $\int_1^4 [x] \, dx$.
- ⑮ Evaluate $\int_{-3}^4 |x| \, dx$.

5 Marks

- ① Prove that $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + C$
- ② Prove that $\int \frac{dx}{\sqrt{x^2+a^2}} = \log|x+\sqrt{x^2+a^2}| + C$
- ③ Evaluate $\int \frac{dx}{\sqrt{2-4x+x^2}}$
- ④ Evaluate $\int \frac{2x+1}{\sqrt{x^2+2x-1}} dx$ and $\int \frac{x+5}{\sqrt{x^2+6x-7}} dx$.
- ⑤ Evaluate $\int \frac{\cos \theta d\theta}{\sin^2 \theta \sqrt{\cos^2 \theta - 4}}$
- ⑥ Evaluate $\int \frac{1}{\sin^2 \theta \sqrt{\cot^2 \theta + 2}} d\theta$
- ⑦ Evaluate $\int \tan^{-1} x dx$.
- ⑧ Evaluate $\int \ln(x^2+x+2) dx$ and $\int (\log x)^2 dx$.
- ⑨ Evaluate $\int x \tan^{-1} x dx$.
- ⑩ Evaluate $\int e^x \left(\frac{1+\sin x}{1+\cos x} \right) dx$.
- ⑪ Evaluate $\int e^x \left(\frac{1+x \log x}{x} \right) dx$.
- ⑫ Evaluate $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$.
- ⑬ Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$.
- ⑭ Evaluate $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$.
- ⑮ Evaluate $\int_0^{\pi/2} \log \tan x dx$

10 Marks

1) Evaluate $\int \sqrt{a^2 - x^2} dx$.

2) Evaluate $\int e^{3x} \cos 2x dx$.

3) Evaluate $\int \ln(x + \sqrt{x^2 + a^2}) dx$.

4) Prove that $\int_0^{\pi/4} \log(1 + \tan \theta) d\theta = \frac{\pi}{8} \log 2$

5) Prove that $\int_0^{\pi/2} \log(\sin x) dx = -\frac{\pi}{2} \log 2$

6) Find the whole area of the circle $x^2 + y^2 = a^2$

-: Differential Equations! -

Q.1 find the order and degree.

(i) $\left(\frac{dy}{dx}\right)^2 + 3y^2 = 5x$ (ii) $\frac{d^2y}{dx^2} = \sqrt{3 + \frac{dy}{dx}}$

(iii) $\frac{dy}{dx} = \frac{3}{\frac{dy}{dx}}$ (iv) $\sqrt{1-y^2} dx + y \sqrt{1-x^2} dy = 0$

(v) $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{5/2} = 3 \left(\frac{d^2y}{dx^2}\right)$

Q.2 (i) find the diff. eqⁿ of the family of curves.

$$y = A \cos x + B \sin x.$$

(ii) find the diff eqⁿ of the family of curves.

$$y = A e^{2x} + B e^{-3x}.$$

Q.3 solve $(1+x^2) dy + (1+y^2) dx = 0$

Q.4 solve $x(1+y^2) dx + y(1+x^2) dy = 0$

Q.5 solve $e^x \tan y dx + (1+e^x) \sec^2 y dy = 0$

Q.6 solve $\sec x \tan y dx + \sec^2 y \tan x dy = 0$

Q.7 solve $\frac{dy}{dx} + (\sec x) y = \tan x.$

Q.8 solve $(1+x^2) \frac{dy}{dx} + 2xy - x^3 = 0$

Q.9 solve $x\left(\frac{dy}{dx}\right) + 3y = x^2$