Aakash Rana
Mechanical

CHAPTER-1

SIMPLE MECHANISM.

Machine-

Machine is a device which receives energy and transforms it into some useful work.

Link-

Each part of a machine, which moves relative to some other part is known as link. It is also known as kinematic link or element. A link should have following characteristics.

i. It should have relative motion.
ii. It must be a resistance body.

Links are 3 types.
a. Rigid link.
b. Flexible link.
c. Fluid link.

Kinematic Pair-

Two links or elements of a machine when in contact with each other is called pair.

If the relative motion exists in between them is called kinematic pair.

Kinematic chain-

When the kinematic pairs are co[illegible] in such a way that the last link [illegible] connected to the first link to transmi[illegible] definite moti[illegible] it is called kinemati[illegible] chain.

It is also defined as the combination of kinematic pair, if the number of pair is 'p' and if the the number of link is 'l' then relation between them is,

$$\boxed{l = 2p - 4}$$

If the number of joints as said to be j, then mathematically,

$$\boxed{j = \frac{3}{2}l - 2}$$

Mechanism—

In a kinematic chain of one link is fixed then it is called mechanism. In a kinematic chain with four links is called simple mechanism. In a kinematic chair with more than four links is said to be compound mechanism.

Classification of kinematic pair—

1. According to the type of relative motion of the link.
2. According to the type of surface contact between the link.
3. According to the type of encloser.

Imp. * According to the type of surface contact between the link is classified in-to two types.

1. Lower pair.
2. Higher pair.

1. Lower pair—

When the two elements or link of a pair have a surface contact while relative motion takes place and the surface of one elements or links slides over the surface of another, the pair formed is known as Lower pair.

Ex—Sliding pair, turning pair, piston cross head.

II. Higher pair—

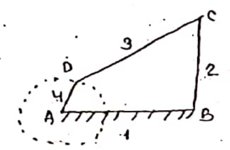
When the two elements of a pair have a line or point contact while in motion, the pair so formed is known as higher pair.

Ex—Belt, rope and chain drive.

Inversion mechanism—

We have already discussed that when one of link is fixed in a kinematic chain, it is called a mechanism. So we can obtain as many mechanisms as the number of links in a kinematic chain by fixing in turn different links. This method of obtaining different mechanisms by fixing different links in a kinematic chain is known as inversion mechanism.

Four bar link mechanism/Quadric cycle chain mechanism—



We have already discussed that the kinematic chain is a combination of four or more kinematic pairs such that the relative motion between the links or elements is completely constrained. The simples and the basic kinematic chain is a four bar chain or quatric cycle chain. From the figure it consists of four links A, B, C and D.

According to Grashof's law the sum of shortest and longest link lengths should not be greater than the sum of remaining two link lengths, if there is to be continuous relative motion between the two links.

A very important consideration in designing of a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful.

In a four bar chain, one of the links, in perticular the shorted link will make a complete revolution relative to the other three, if it satisfies Grashof's law such a link is known as crank. From figure, the shortest

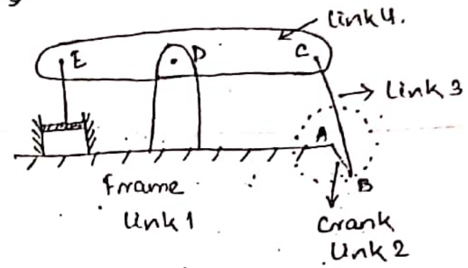
link AD → Crank
BC → Liver
AB → Frame
CD → connecting rod.

Inversion of four bar link mechanism—

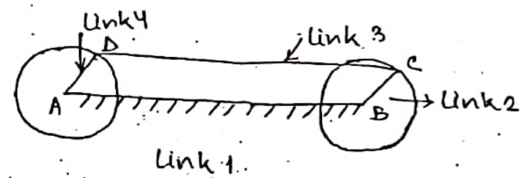
The following are the important in inversion of four bar link mechanism.

I. Beam engine.
II. Coupling rod of a locomotive.
III. Watt's indecator mechanism.

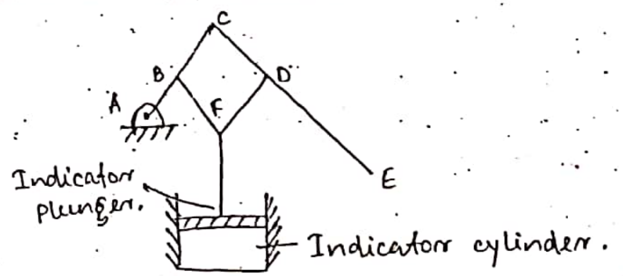
I. Beam engine—



II. Coupling rod of a locomotive—



III. Watt's indicator mechanism—



From the figure the link CE and BFD acts as livers. The displacement of the link BFD is directly proportional to the pressure of gas or stream, which acts on the indicator plunger. On any small displacement of the mechanism the tracing point E at the end of the link CE traces out approximately a straight line.

Beam engine–

1. Here, when the crank rotates about the fixed centre A the liver oscillates about the fixed centre D.
2. The end E of CDE is connected to the piston which reciprocates as a result.
3. The purpose of this mechanism is to transmit the rotary motion into reciprocating motion.

Coupling rod of a locomotive–

I. In other words it is known as double crank mechanism.
II. From the figure the link AB is fixed and AD = BC. Both crank are connected to the respective wheels.
III. The link CD is called connecting rod.
IV. The purpose of this mechanism is to transmitting the rotary motion from one wheel to another.

Cams & Followers–

I. Cam and followers are the rotating machine elements which gives the reciprocating or oscillating motion to another element which is the follower.
II. Cam and followers have line contact and they constitute a higher pair.
III. Cams usually are rotated at uniform speed by a shaft but the motion of the follower is pre-determined by the shape of the cam.
IV. Cam finds its applications in the inlet and exhaust valves up an IC engine.
V. When produced in large scale cams are manufactured by a punch press, die casting, milling from the master cam.

Classification of followers–

Followers are classified into 3 types.

1. Surface in contact.
2. Motion of the follower.
3. Path of the motion of the follower.

According to surface in contact–

Followers are further classified into following categories–

1. Knife edge follower–
   I. The contancting edge of the follower has sharp knife edge.
   II. The sliding motion takes place between the knife edge of the follower and the surface of the cam.
   III. This type of follower is seldom used in practice as a small area of contacting surface.
   IV. Large amount of side thrust exist between the follower and the guide.

2. Roller follower–
   I. The contacting end of the follower is a roller and rolling motion takes place between the contacting surfaces of followers and cam. So that the rate of wear is greedely reduced.
   II. But side thrust exist between the follower and the guide. It is extensively used where more space is available

III. These type of followers are mostly used in oil engines, air craft engines, stationary gas, oil engines etc.

3. Flat faced followers—

I. The contacting end of the follower is a perfectly flat faced.

II. Side thrust between the follower and guide is much reduced. But only side thrust due to friction between the contacting surface of the followers and cam exists.

III. Relative motion between these surfaces is largely sliding in nature. But where may be reduced by off setting the axeses of the follower.

IV. These type of follower is used where space is limited such as the cams that operates the valves of the automobile engine.

4. Spherical faced surface—

I. When the flat faced ~~surface~~ follower is used in automobile high surface stress are produced and hence to minimize the stress spherical faced is used.

II. Here the contacting end of the follower has a spherical faced edge.

According to the motion of the follower—

According to the motion of the follower it is classified into two types.

1. Reciprocating / Translating follower—

When the follower reciprocates in the guides ~~when~~ as the cam rotates uniformly. Then it is called reciprocating / translating follower.

2. Oscillating / Rotating follower—

When the uniform rotary motion of the cam is converted into predetermined oscillating motion of the follower, then it is called oscillating follower.

According to the path of the motion of the follower—

Radial follower—

When the motion of the follower is along the axis passing through the centre of the cam then it is known as a radial follower.

Offset follower—

When the motion of the follower is along the axis away from the axis of the cam centre then it is known as offset follower.

Classification of cams—

Cams are classified into 2 types.

1. Radial or disc cam—

I. In radial cam follower reciprocates or oscillates in a direction perpendicular to the cam.

II. Radial cams can be reciprocating type, tangent type or circular type.

2. Cylindrical ~~type~~ cam—

Here the follower reciprocates or oscillates in a direction parallel to the cam axeses.

CHAPTER-3

POWER TRANSMISSION.

→ From one shaft to another by the help of pulleys which rotates at the same and different speed. This can be transmitted by the help of belt or chain or ropes.

→ The amount of power transmitted depends upon

1. Velocity of the belt.
2. Tension under which the belt is placed on the pully.
3. The arc of contact between the belt and the smaller pulley.
4. Conditions under which the belt is used.

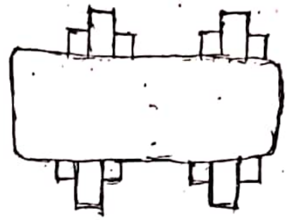
→ Depending upon the amount of power transmitted belt drives are of following drive.

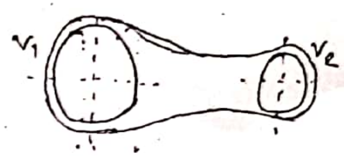
1. Light drive – This type of drive transmit less power and the speed of rotation is limited upto 10 m/sec.
2. Medium drive – This type of drive is used a relatively larger power then light drives but the speed is limited between 10 – 22 m/sec
3. Heavy drive – This type of drive used to transmit large amount of power and the speed goes beyond 22m/sec.

→ Depending upon the shape the belts are of following type.

1. Flat belt – Flat belts are used when moderate amount of power is to be transmitted and the distance between the two pulleys are not more than 8 mtrs.
2. V-Belt – This type of belt also transmit medium amount of power but two pulleys are very near to each other.
3. Circular belt – When great amount of power is to be transmitted and the pullies are more than 8m apart then circular belt are used. Circular or rope belt.

Velocity ratio of belt drive–





Let d_1 = diameter of the driver pulley
d_2 = diameter of the follower pulley
N_1 = Speed of the driver (rpm)
N_2 = Speed of follower (rpm)

In one minute. the length of the belt that passes over the driver is equal to $\pi d_1 \times N_1$.
Similarly for follower length of the belt $\pi d_2 N_2$.
Since length of the belt that passes over the driver in one minute is equal to the length of the belt that passes over the follower in the same time; So,

$$\pi d_1 N_1 = \pi d_2 N_2$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

So the velocity ratio $\frac{N_2}{N_1} = \frac{d_1}{d_2}$

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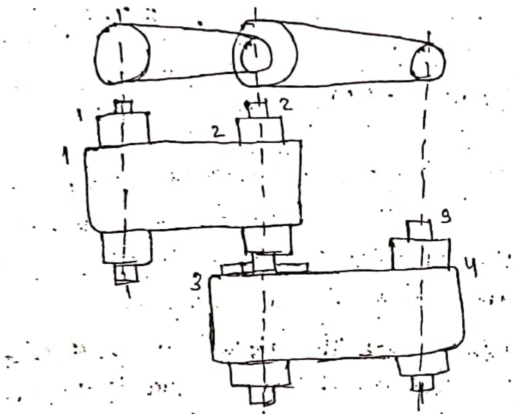
If the thickness of the belt is considered then $\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}$

If the peripheral velocity is to be calculated then

$$\frac{V_2}{V_1} = \frac{d_1}{d_2}$$

Velocity ratio of compound belt drive.



Let d_1, d_2, d_3, d_4 be the diameter of the pulley 1, 2, 3, 4 respectively.

N_1, N_2, N_3 & N_4 are the speed of pulley 1, 2, 3 & 4 respectively.

∴ The velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \cdots (1)$$

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \quad \cdots (2)$$

Multiplying both the eqⁿ

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

Since the pulley 2 & 3 are on the same shaft so $N_2 = N_3$ (∵ Attached to each other).

$$\frac{N_4}{N_1} = \frac{d_1 d_3}{d_2 d_4}$$

If the no. of pulleys is taken to be six then

$$\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}$$

Generally, $\frac{\text{speed of last driven}}{\text{speed of first driver}} = \frac{\text{Product of driver dia.}}{\text{Product of driven dia.}}$

2) Slip of belts –

→ The functional grip between the belt and the pulley or shaft some times become in sufficient which may cause forward motion of the driver without carrying the ~~bet~~ belt without carrying the driven pulley. This is called as the slip of the belt.

→ Generally slip reduces the velocity ratio of the system.

Let $s_1 \%$ = slip betⁿ the driver & belt.

$s_2 \%$ = slip betⁿ the belt & follower.

The velocity of the belt passing over the driver is

$$V = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \cdot \frac{s_1}{100}$$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)$$

The velocity of the follower after considering slip of the belt is

$$\frac{\pi d_2 N_2}{60} = V - \frac{V \cdot s_2}{100}$$

$$\Rightarrow \frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_2}{100} - \frac{s_1}{100} + \frac{s_1 s_2}{10000}\right)$

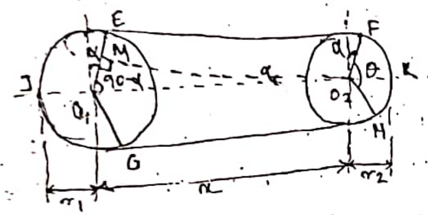
$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1+s_2}{100}\right)$

when along with the slip of the belt the thickn is also consider.

$$\frac{N_2}{N_1} = \frac{d_1+t}{d_1+t}\left(1 - \frac{s_1+s_2}{100}\right)$$

Length of the belt drive –

Case-1 (Open belt drive)



Let r_1 & r_2 be the radius or larger and smalle pulley.

x = distance betn the centre of two pulleys

L = The total length of the belt

Length of the belt, $L = EF + FKH + HG + GJE$
$= 2(\text{Arc } JE + EF + \text{Arc } FK)$

A line parallel to EF is drawn having an ang α at O_2.

$\sin\alpha = \frac{O_1M}{O_1O_2}$

$= \frac{r_1 - ME}{O_1O_2} = \frac{r_1 - r_2}{x}$

as α is a very small quantity so $\alpha = \sin\alpha = \frac{r_1 - r_2}{x}$

Arc $JE = r_1\left(\frac{\pi}{2}+\alpha\right)$

Arc $FK = r_2\left(\frac{\pi}{2}-\alpha\right)$

$EF = MO_2 = \sqrt{O_1O_2^2 - MO_1^2}$ (By pythagorious theorem)

$= \sqrt{x^2 - (r_1 - r_2)^2}$

$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$

$= x\left[1 - \left(\frac{r_1 - r_2}{x}\right)^2\right]^{1/2}$

$= x\left[\left(1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2\right) + \cdots\right]$

$\Rightarrow EF = x - \frac{(r_1 - r_2)^2}{2x}$

So using the values of arc JE, EF & FK. the length of the belt.

$L = 2\left[r_1\left(\frac{\pi}{2}+\alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2}-\alpha\right)\right]$

$L = 2\left[r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2\frac{\pi}{2} - r_2\alpha\right]$

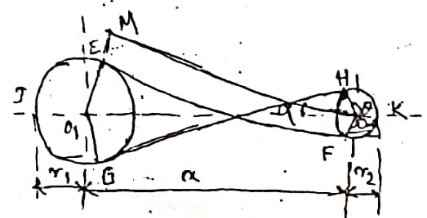
$L = 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$

$L = 2\left[\frac{\pi}{2}(r_1 + r_2) + \frac{(r_1 - r_2)}{x}\cdot(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$

$= \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x$

Or $L = \frac{\pi}{2}(d_1 + d_2) + \frac{(d_1 - d_2)^2}{4x} + 2x$

Case-2 (Cross type).



The length of the belt for a cross belt pulley is given by, L = arc $GJE + EF + HG + FKH$ Arc

$\Rightarrow L = 2(\text{Arc } JE + EF + \text{Arc } FK)$

Suppose a parallel line is drawn to EF passing through the centre O_2' and inclined at an angle α.

Arc $JE = r_1(\pi/2 + \alpha)$

$FK = r_2(\pi/2 + \alpha)$

In the $\triangle O_1MO_2$

$$\sin\alpha = \frac{O_1N}{O_1O_2} = \frac{OE + EM}{O_1O_2}$$

$$= \frac{r_1 + r_2}{x}$$

As α is a small quantity so $\sin\alpha = \alpha = \frac{r_1 + r_2}{x}$

$$EF = MO_2 = \sqrt{O_1O_2^2 - O_1M^2}$$

$$= \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

$$= x\left(1 - \frac{r_1 + r_2}{x}\right)^{1/2} = x - \frac{(r_1 + r_2)^2}{2x}$$

(By applying Binomial theorem)

$$L = 2\left(r_1\left(\frac{\pi}{2} + \alpha\right) + x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2} + r_2\left(\frac{\pi}{2} + \alpha\right)\right)$$

$$= 2\left(r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2\frac{\pi}{2} + r_2\alpha\right)$$

$$= 2\left(\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right)$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2\frac{r_1 + r_2}{x}(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

(Putting $\alpha = \frac{r_1 + r_2}{x}$)

$$L = \pi(r_1 + r_2) + \frac{(r_1 + r_2)^2}{x} + 2x$$

$$\Rightarrow L = \frac{\pi}{2}(d_1 + d_2) + \frac{(d_1 + d_2)^2}{4x} + 2x$$

Power transmitted by the belt –

The tension in the tight side of the belt is taken to be T_1 and the tension in the slack side is taken to be T_2 so the total tension in the belt is equal to $T = T_1 - T_2$.

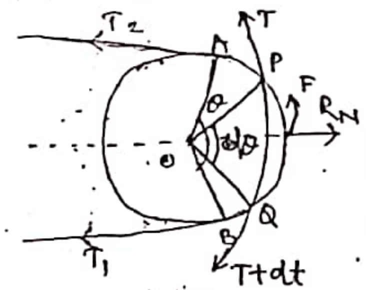
So the power transmitted by the belt is taken to be, $P = T \times V$, where V = velocity of the belt

$$\Rightarrow P = (T_1 - T_2)V$$

Ratio of driving tension for flat belt drive –

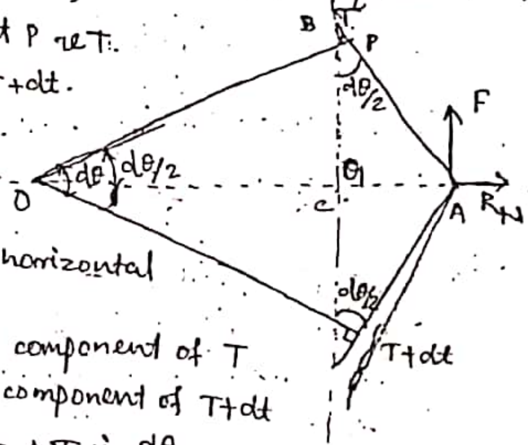
Let t_1 & t_2 be the tensions in the tight and slack side respectively.

θ = angle of contact through which the belt touches the pulley.



Considering a small portion PQ having an angle $d\theta$ at the centre the following forces are in equillibrium.

1. The tension at point P ie T.
2. Tension at point ie T+dt.
3. Normal reaction R_N.
4. Frictional force F.



Resolving the forces in horizontal direction

R_N = The horizontal component of T + horizontal component of T+dt

$$\Rightarrow R_N = (T + dt)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2}$$

Since, $\frac{d\theta}{2}$ is very small component $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$

$$\Rightarrow R_N = (T + dt)\frac{d\theta}{2} + T\frac{d\theta}{2}$$

$$= 2T\frac{d\theta}{2} + dt \cdot \frac{d\theta}{2}$$

$= Td\theta + dt\frac{d\theta}{2}$

$R_N = Td\theta$

Neglecting $dt.d\theta$, we get $R_N = Td\theta$

Resolving the forces in vertical direction,

$F =$ vertical component of $T -$ vertical comp. T

$= T + dt\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2}$

since $\frac{d\theta}{2}$ is very small quantity $\cos\frac{d\theta}{2} = 1$.

$F = T + dt - T$

$F = dt$

$\Rightarrow \mu R_N = dt$

$\Rightarrow \mu Td\theta = dt$

$\Rightarrow \mu d\theta = \frac{dt}{T}$

$\mu\int_0^\theta d\theta = \int_{T_2}^{T_1}\frac{dt}{T}$

$\Rightarrow \mu\theta = [\ln T]_{T_2}^{T_1}$

$\Rightarrow \mu\theta = \ln\left(\frac{T_1}{T_2}\right) = 2.333\log\left(\frac{T_1}{T_2}\right)$

$\Rightarrow e^{\mu\theta} = \frac{T_1}{T_2}$

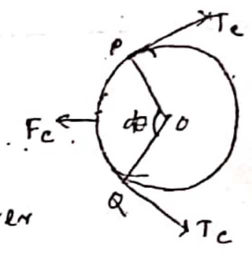
Centrifugal tension—

Since the belt continuously runs over the pulleys therefore some centrifugal force is called whose effect is to increase the tension on the both tight and slack side.

→ The tension caused by this centrifugal force is called centrifugal tension.

→ When the speed of the pulley is low the speed is less then 10m/sec. Then the centrifugal force is very small and thus not taken into account.

→ But when the speed is higher i.e. more than 10m/sec than the centrifugal tension is taken into consideration.



Let m = mass of the belt per unit length.

v = Linear velocity of the belt.

r = Radius of the pulley over which the belt runs.

T_c = Centrifugal tension acting tangentially

When a small portion PQ is considered having an angle $d\theta$ at the centre of the pulley then the arc $PQ = r.d\theta$

Mass of $PQ = m \times$ length at $PQ = mrd\theta$

Centrifugal force due to PQ is

$F_C = mrd\theta \times \frac{v^2}{r}$

$= md\theta \cdot v^2$

Resolving the forces in horizontal & vertical direction the vertical direction forces cancel out each other and the horizontal components are balanced by the centrifugal force.

$F_c = 2.T_c.\sin\frac{d\theta}{2}$

$= 2T_c \cdot d\theta/2$ (since $\frac{d\theta}{2}$ is a small quantity)

$= T_c d\theta$

$\Rightarrow md\theta \cdot v^2 = T_c d\theta$

$\Rightarrow T_c = mv^2$

The total tension in the tight side is equal to

$T_{t_1} = T_1 + T_c$

The total tension in the slack side is $T_{t_2} = T_2 + T_c$

The power transmitted, $P = (T_1 - T_2)V$

$$P = (T_{t_1} - T_c - T_{t_2} + T_c)V$$

$$P = (T_{t_1} - T_{t_2})V$$

Also considering the centrifugal tension in the ratio of driving tensions will give

$$\ln\left(\frac{T_{t_2} - T_c}{T_{t_1} - T_c}\right) = \mu\theta$$

Initial tension in the belt –

When the pulleys are stationary the belt is subjected to some tension as the ends of the belt are ~~continuously~~ join together so that when the shaft rotates the belt is continuously moved over the pulley this tension is called as the initial tension of the belt.

→ For the tight side of the belt is initial tension reduces the total tension of that side and for the slack side the initial tension also tends to ∴ decrease the total tension of that side.

Let T_1 = tension in the tight side of the belt.

T_2 = tension in the slack side in the belt.

T_0 = Initial tension of the belt.

The total tension in the tight side $T_t = T_1 - T_0$

slack side $T_s = T_0 - T_2$

Let α is the co-efficient of increase of the belt length per unit force.

The expansion of the belt in the tight side is αT_t or $\alpha(T_1 - T_0)$ change in length of the belt in the slack side is $\alpha T_s = \alpha(T_0 - T_2)$.

But as the length of the belt passing over the pulley so the changes of belt length in the tight side is to be equal to the changes in the belt length of the slack side.

$$\Rightarrow \alpha(T_1 - T_0) = \alpha(T_0 - T_2)$$

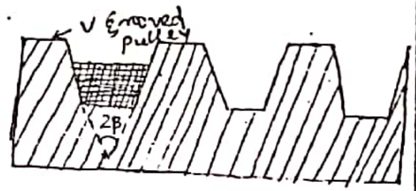
$$\Rightarrow T_1 - T_0 - T_0 + T_2 = 0$$

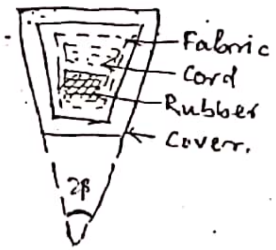
$$\Rightarrow -2T_0 = -T_1 + T_2$$

$$\Rightarrow T_0 = \frac{T_1 + T_2}{2}$$

When the speed of the rotation of the shaft is large enough to consider the centrifugal tension, $T_0 = \dfrac{T_1 + T_2 + 2T_c}{2}$

V – Belt drive –



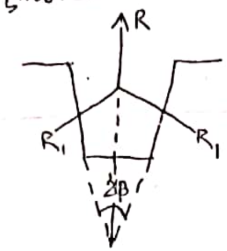


I. V belts are made up of fabric and cords molded in rubber and covered with a layer of fabric and rubber.

II. The belts are moulded to a trapezoidal shape whose angle is 30°–40°.

III. In case of flat belt pulley the belt runs over the pulley where as for the case of V-belt drive the rim of the pulley is grooved in a V-shape. So that the V-belt runs over it.

iv. The effect of the groove is to increase the frictional grip of the V-belt and thus to reduce the tendency of sliping. Let R is the total reaction in the plane of the groove.



R_1 = The normal reaction between the belt & side of the groove.

2β = Angle of the groove.

μ = co-efficient of friction.

Resolving the vertical and horizontal reaction the horizontal reaction ~~character~~ cancel out each other for the vertical.

$$R = 2(R_1 \sin\beta)$$

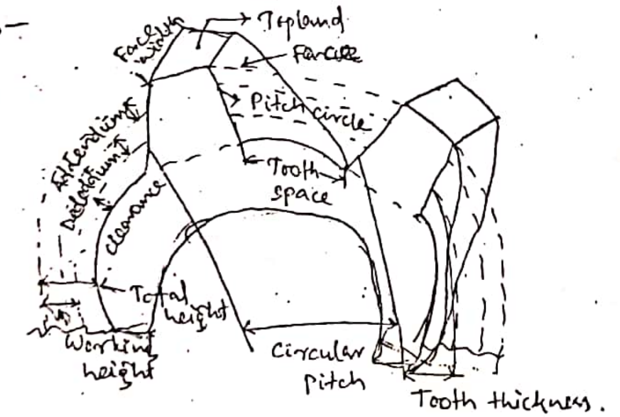
$$R_1 = \frac{R}{2\sin\beta}$$

The frictional force; $F_r = 2\mu R_1$

$$= 2\mu \frac{R}{2\sin\beta} = \frac{\mu R}{\sin\beta} = \mu R \operatorname{cosec}\beta$$

For the case of V-belt the ratio of driving tension is, $\ln\left(\frac{T_2}{T_1}\right) = \mu\theta \operatorname{cosec}\beta$.

Gears—



There are various terminology associated with gears. They are as follows—

1. Pitch circle— It is an imaginary circle which by pure rolling action will give the same motion as the actual gear.
2. Pitch circle diameter— It is the diameter of the pitch circle whose generally specifies the size of the gear.
3. Pitch point— It is the common point of contact between two pitch circles.
4. Pitch surface— It is the surface of the rolling disc which the moshing gears have replaced at the pitch circle.
5. Pressure angle— It is the angle between the common normal to two gears teeth at the point of contact and common tangent. The standard pressure angles are $14\frac{1}{2}°$ or $20°$.

6. Addendum—

It is the radial distance from the pitch circle to the top of the teeth.

7. Dadendum—

It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. Circular pitch—

It is the distance measured on the circumference of the pitch circle from a point of one teeth to the corresponding point on the next tooth. It is denoted as P_c.

$$P_c = \frac{\pi D}{T}$$

where, D = Diameter of the pitch circle.
T = No. of teeth on the wheel or gear.

9. Diametrical pitch–
It is the ratio of no. of teeth to the pitch circle diameter in mm.
It is denoted as Pd.
Mathematically, $P_d = \frac{T}{D}$

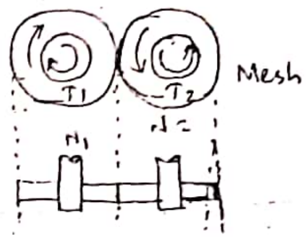
10. Module–
It is the ratio of pitch circle diameter in mm. to the no. of teeth. It is denoted as m.
Mathematically, $m = \frac{D}{T}$.

Gear trains–
When two or more gears are made to mesh with each other to transmit power from one shaft to another such a combination is called a gear train.

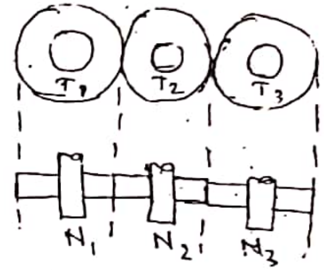
Types of gear trains–
1. Simple gear train
2. Compound gear train
3. Reverted gear train.
4. Epicyclic gear train.

1. Simple gear train–
When there is one gear on each of the shafts then it is known as a simple gear train.
Let N_1 = Speed of the gear 1 (rpm)
N_2 = Speed of gear 2 (rpm)
N_3 = Speed of gear 3 (rpm)

T_1 = No. of teeth on gear 1
T_2 = No. of teeth on gear 2
T_3 = No. of teeth of gear 3



The speed ratio of the gear train, when only the two shafts are consider is $\frac{N_1}{N_2} = \frac{T_2}{T_1}$



But when the no. of shafts are increased let say to 3
then, $\frac{N_1}{N_2} = \frac{T_2}{T_1}$ & $\frac{N_2}{N_3} = \frac{T_3}{T_1}$

$\Rightarrow \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$

$\Rightarrow \frac{N_1}{N_3} = \frac{T_3}{T_2}$

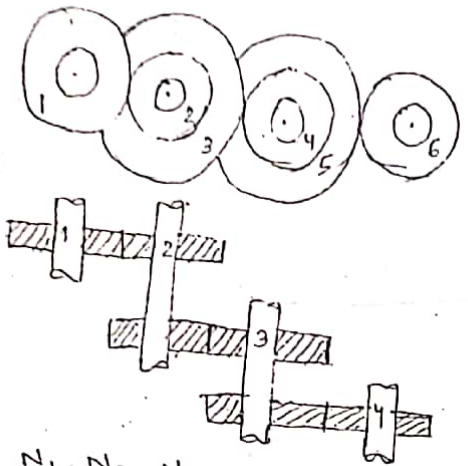
Similarly if 'n' no. of gears are taken then,

$$\boxed{\frac{N_1}{N_n} = \frac{T_n}{T_1}}$$

$$\frac{\text{Speed of first driver}}{\text{speed of last follower}} = \frac{\text{No. of teeth on last follower}}{\text{No. of teeth of first driver.}}$$

When the no. of gears are add in number then the motion of the first driver and the last follower is like (similar in direction). but when the no. of gears are even in no. then the motion is transmitted from driver to follower is opposite in nature.

2. Compound gear train–
When there are more than one gear on the shafts it is called as a compound gear train.



Let N_1, N_2, N_3, N_4, N_5 & N_6 = speed of gear 1,2,3, 4,5 & 6 respectively,

T_1, T_2, T_3, T_4, T_5 & T_6 = No. of teeth on gear 1, 2, 3, 4, 5 & 6 respectively.

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}, \quad \frac{N_3}{N_4} = \frac{T_4}{T_3}, \quad \frac{N_5}{N_6} = \frac{T_6}{T_5}$$

As gear 2 & gear 3 lying on one shaft 2. So $N_2 = N_3$.

Similarly, $N_4 - N_5$

Then, $\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$

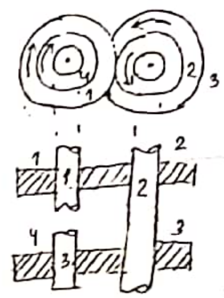
$$\Rightarrow \frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5} \quad (\because N_2 = N_3 \ \& \ N_4 = N_5)$$

The advantage of a compound train over a simple gear train is that a much large speed is either reduced or increased from the first shaft to the last shaft.

When the speed is transmited from a similar gear to a larger-1 then there is a speed reduction and vice versa and when vice versa is used then there is a speed increment.

3. Reverted gear train –

→ When the axes of the first gear and the last gear are co axial then it is known as a reverted gear train.



Let N_1, N_2, N_3 & N_4 = speed of gear 1, 2, 3, 4 respectively.

T_1, T_2, T_3 & T_4 = No. of teeth or 1, 2, 3, 4

r_1, r_2, r_3 & r_4 = radius of gear 1, 2, 3 & 4 respectively.

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\frac{N_3}{N_4} = \frac{T_4}{T_3}$$

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} = \frac{T_2 \cdot T_4}{T_1 \cdot T_3}$$

$$\Rightarrow \boxed{\frac{N_1}{N_4} = \frac{T_2 T_4}{T_1 T_3}} \quad (\because N_2 \ \& \ N_3 \text{ are on same shaft, so } N_2 = N_3)$$

Also, as the centre distance of shaft 1 & 2 is equal to 2 & 3.

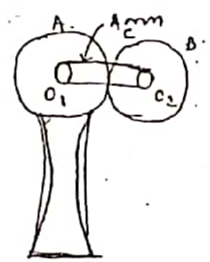
So $r_1 + r_2 = r_3 + r_4$

$\Rightarrow d_1 + d_2 = d_3 + d_4$

Form this it can be concluded that,

$$T_1 + T_2 = T_3 + T_4.$$

4. Epicyclic gear train –



When gear A and arm C have a common axis at O_1 about which they rotate similarly the gear B has its axis at O_2 and the gear B meshes with gear A.

If the arm C is fixed the gear train is simple and gear A drives gear B and vice versa.

But if the A is fixed and arm C rotates about O_1 then the gear B is force to rotate upon and arround gear A such a motion is called epicyclic and (the gear train arranged in such a manner that one or more of their members rotate upon and arround another member then it is known as epicyclic gear train)

An epicyclic gear train may be simple gear train or compound gear train.

Maximum tension–

$T_{max} = T_1 + T_c$ (For pulley)

$\Rightarrow T_{max} = \sigma b t$ (For belt)

where σ_b is the working stress of the belt.

b is the width of the belt.

t is the thickness of the belt.

2 ⇒ The function of a governor is to regulate the mean spaced of a engine when there are variation in the load on the engine when the load on the engine increases the speed of the engine decreses for which it is necessary to increase the amount of working fluid and on the other hand when the load of the engine decreases its speed increases and hence the amount of working fluid is required decreases.

⇒ In such case the governor automatically controls the supply of working fluid to the engine with varrying load condition and keep the mean speed of the engine with in certain limit.

Types of Governors–

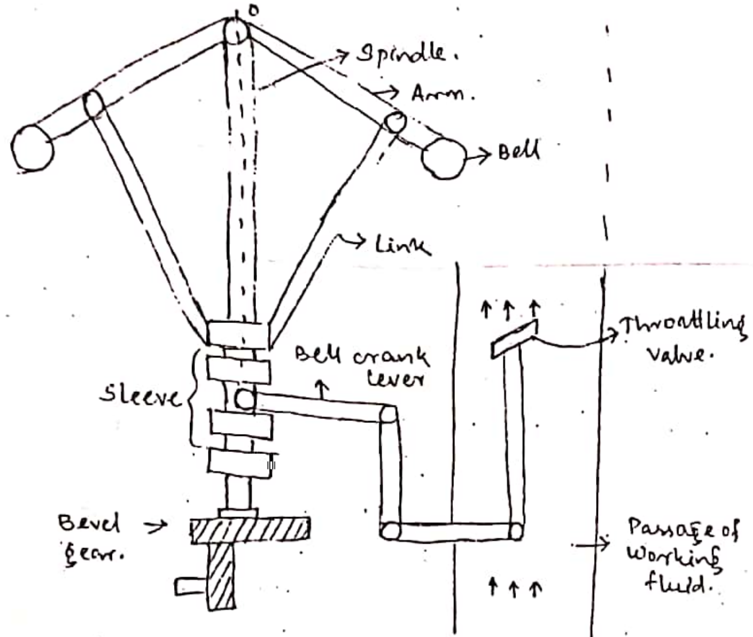
Governors are generally classified into two types.

1. Centrifugal.
2. Inertia.

The centrifugal governor is further classified into the following types.

1. Pendulum type. (Watt governor)
2. Loaded type
   a. Dead weight – Porter, Proell
   b. Spring control – Hartnell, Harting, Willson Hartnell, Pickering.

Centrifugal Governor–



The centrifugal governor are based on the balancing of the centrifugal force on the rotating bulbs by an equal and opposite radial force known as controlling force.

A centrifugal governor consists of two balls of equal mass attached to the arms of the governor. These bulbs are called governor bulbs or fly bulbs.

Case-I –

When the load on the engine increases the speed of engine and speed of fly balls decreases. This results in decrease of centrifugal force on the ball and hence the bulb moves inwards and the sleeve of the engine moves downwards. The downward movement of the sleeve operates a

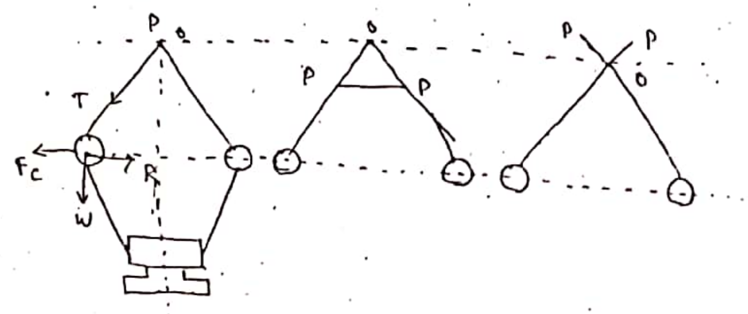
throttle valves at the other end of the belt crank lever to increase the supply of working fluid and thus the speed of the engine is increased.

In this case the ~~extra~~ power output is provided to balance the increasing load.

Case-II –

When the load on the engine decreases the engine as well as the fly balls speed increases and thus the centrifugal force acting on the bulbs also increases. Due to this, the balls moves outwards and the sleeves rises upwards this upward movement of the sleeves reduces the supply of working fluid and hence the speed of the engine is decreased. In this case the power output is reduced.

Watt Governor –



The simplest form of centrifugal governer is the watt governor which is basically a conical pendulum with links attached to a sleeve of negligible mass.

→ The arms of the governor may be connected to the spindle in three following ways –

1. The pivot P may be on the spindle axis.
2. The pivot P may be on offset from the spindle axis and the arms when produced intersect at 'O'.
3. The pivot P may be at offset from the spindle axis but the arms cross at 'O'.

In this case, let m = mass of the fly ball (kg)

w = weight of the fly ball $= mg$ (N)

T = Tension in the arm (N)

ω = Angular velocity of the ball & the arm about spindle axis (rad/sec).

r = radius of path or rotation of the fly balls from the centre is spindle axis (m).

F_c = Centrifugal force (N)

h = height of the axis of fly ~~wheel~~ ball from point is, or also called as the governors height.

It is assumed that the weight of the arm link and the sleeve are negligible as compare to the weight of the ball. So the ball is in equillibrium under the action of F_c, w & T.

Considering moment at point 'O',

$F_c \times h = w \times r$

$\Rightarrow m r \omega^2 \times h = w \times r$

$\Rightarrow m r \omega^2 \times h = m g r$

$\Rightarrow \omega = \sqrt{\frac{g}{h}} \Rightarrow h = g/\omega^2$.

Also $\omega = \frac{2\pi N}{60}$, where, N = speed in rpm.

$\Rightarrow h = \frac{g \cdot 60^2}{(2\pi N)^2}$

$h = \frac{895}{N^2}$

or $N = \sqrt{\frac{895}{h}}$

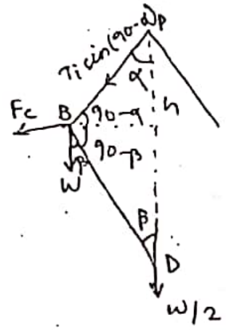
The speed of fly ball is only depended on the height of the governor.

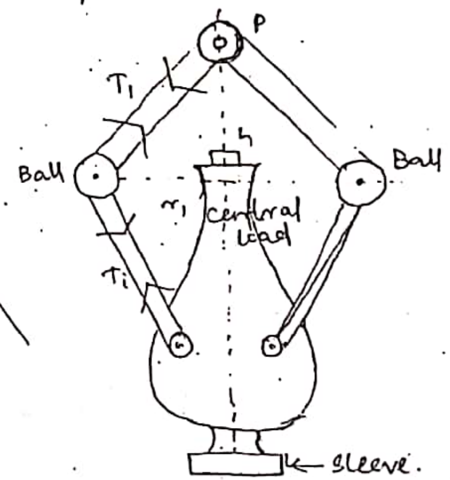
Imp

Porter governer –

→ The porter governer is the modification of watt governer with a central load attached to the sleeve of the governor.

→ In this case also the loads move up and down the central spindle.





Let, m = mass of the fly ball (kg)

w = weight of the fly ball (N)(mg)

M = Mass of central Load

W = weight of the central Load (N)(Mg)

r = radius of rotation of the fly ball from the spindle.

h = height of the governor.

T_1, T_2 = The tension in the arm and link respectively.

α = Angle of inclination of the arm to the vertical axis.

β = Angle of inclination of the link to the vertical axis.

The speed of the governor can be determined by two methods.

1. Method of resolving forces—

At point D resolving the vertical forces

$$T_2 \cos\beta = \frac{W}{2}$$

$$\Rightarrow T_2 = \frac{Mg}{2\cos\beta} \quad \ldots (1)$$

Considering the equilibrium of forces at point B and resolving the forces in vertical direction,

$$T_1 \cos\alpha = w + T_2 \cos\beta$$

$$= w + \frac{Mg\cos\beta}{2\cos\beta}$$

$$T_1 \cos\alpha = mg + \frac{Mg}{2} \quad \ldots (2)$$

Resolving the forces in horizontal direction

$$F_c = T_1 \sin\alpha + T_2 \sin\beta.$$

$$= T_1 \sin\alpha + \frac{Mg}{2\cos\beta} \cdot \sin\beta.$$

$$F_c = T_1 \sin\alpha + \frac{Mg}{2} \tan\beta$$

$$\Rightarrow T_1 \sin\alpha = F_c - \frac{Mg}{2} \tan\beta \quad \ldots (3)$$

Dividing equation (3) by equation (2)

$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_c - \frac{Mg}{2} \tan\beta}{\frac{Mg}{2} + mg}$$

$$\Rightarrow \tan\alpha = \frac{F_c - Mg/2 \tan\beta}{mg + Mg/2}$$

$$\Rightarrow \left(mg + \frac{Mg}{2}\right)\tan\alpha = F_c - Mg/2 \tan\beta.$$

$$\Rightarrow mg + \frac{Mg}{2} \tan\alpha + M$$

$$\Rightarrow \frac{Mg}{2} + mg = \frac{F_c}{\tan\alpha} - \frac{Mg}{2} \cdot \frac{\tan\beta}{\tan\alpha}$$

As $\tan\alpha = \frac{r}{h}$ and taking $\frac{\tan\beta}{\tan\alpha} = q$

$$\Rightarrow \frac{Mg}{2} + mg = \frac{F_c . h}{r} - \frac{Mg}{2} q.$$

$$\Rightarrow \frac{Mg}{2}(1+q) + mg = \frac{F_c h}{r}$$

$$= \frac{m r \omega^2 . h}{r}$$

$$\Rightarrow h = \frac{mg + \frac{Mg}{2}(1+q)}{m\omega^2}.$$

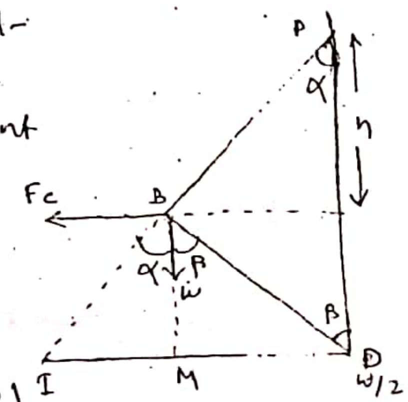
$$\Rightarrow \omega^2 = \frac{mg + \frac{mg}{2}(1+q)}{m . h}$$

$$\Rightarrow h = \frac{M + \frac{m}{2}(1+q)}{m} \times \frac{g \times 60^2}{(2\pi)^2 N^2}$$

$$\Rightarrow h = \frac{m + \frac{M}{2}(1+q)}{m} \cdot \frac{895}{N^2}$$

2. Instantaneous centre method—

Let the line PB & DM be extended to meet at a point I which is the instantaneous center.



$$F_c \times BM = W \times IM + \frac{W}{2} \cdot ID$$

$$\Rightarrow F_c = W \times \frac{IM}{BM} + \frac{W}{2} \frac{ID}{BM}$$

$$= W \times \frac{IM}{BM} + \frac{W}{2}\left(\frac{IM}{BM} + \frac{MD}{BM}\right)$$

$$= W . \tan\alpha + \frac{W}{2}(\tan\alpha + \tan\beta)$$

Dividing eq^n by $\tan\alpha$,

$$\frac{F_c}{\tan\alpha} = w + \frac{W}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)$$

Putting $W = Mg$ & $w = mg$, $\tan\alpha = \frac{r}{h}$,

$$\frac{\tan\beta}{\tan\alpha} = q, \quad F_c = mr\omega^2$$

$$\Rightarrow \frac{mr\omega^2}{r/h} = mg + \frac{Mg}{2}(1+q)$$

$$\Rightarrow m\omega^2 h = mg + \frac{Mg}{2}(1+q)$$

$$h = \frac{1}{m\omega^2} \times \left[m + \frac{M}{2}(1+q)\right] g$$

$$\Rightarrow h = \frac{m + \frac{M}{2}(1+q)}{m} \cdot \frac{g}{\omega^2}$$

$$\Rightarrow \boxed{h = \frac{M + \frac{M}{2}(1+q)}{m} \cdot \frac{895}{N^2}}$$

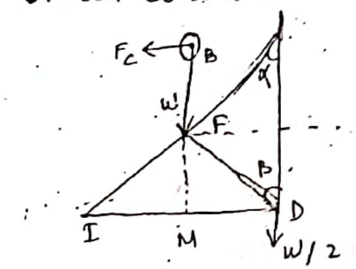
In this case, both the arms & the link the governor are equal then $\alpha = \beta$.

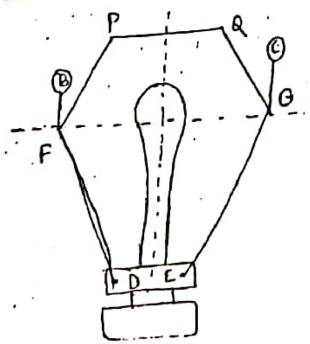
$\Rightarrow \tan\alpha = \tan\beta$

$\Rightarrow q = 1$

Then, $\boxed{h = \frac{M+M}{m} \cdot \frac{895}{N^2}}$

Proell Governor:-





The proell governor has ball fixed at point B & C.

The arms FP & GQ are pivoted at points P & Q respectively also the instantateous center is marked at point I.

Taking moment at point I,

$$F_c \times BM = w \times IM + \frac{W}{2} \times ID$$

Dividing the equation by BM

$$F_c = w \cdot \frac{IM}{BM} + \frac{W}{2} \times \frac{ID}{BM}$$

Multiplying and dividing with FM

$$F_c = w\left(\frac{IM}{BM} \times \frac{FM}{FM}\right) + \frac{W}{2}\left(\frac{ID}{BM} \times \frac{FM}{FM}\right)$$

$$= w\left(\frac{IM}{FM} \times \frac{FM}{BM}\right) + \frac{W}{2}\left(\frac{ID}{FM} \times \frac{FM}{BM}\right)$$

Taking $\frac{FM}{BM}$ common and substituting $\tan\alpha$ & $\tan\beta$.

$$\Rightarrow F_c = \frac{FM}{BM}\left(w\tan\alpha + \frac{W}{2}(\tan\alpha + \tan\beta)\right)$$

Now putting the value of $F_c = mr\omega^2$, $\tan\alpha = \frac{r}{h}$, $\frac{\tan\beta}{\tan\alpha} = q$, $w = mg$ & $W = Mg$.

$$mr\omega^2 = \frac{FM}{BM} \cdot \frac{r}{h}\left(mg + \frac{Mg}{2}(1+q)\right)$$

$$\Rightarrow h = \frac{FM}{BM}\left(\frac{m + \frac{M}{2}(1+q)}{m}\right)\frac{g}{\omega^2}$$

$$= \frac{FM}{BM}\left(\frac{m + M/2\,(1+q)}{m}\right)\frac{895}{N^2}$$

When the length of the link and arm of the governor are equal to, $\alpha = \beta$

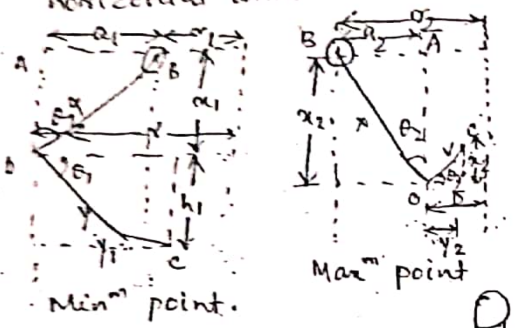
$\Rightarrow \tan\alpha = \tan\beta \Rightarrow q = 1$

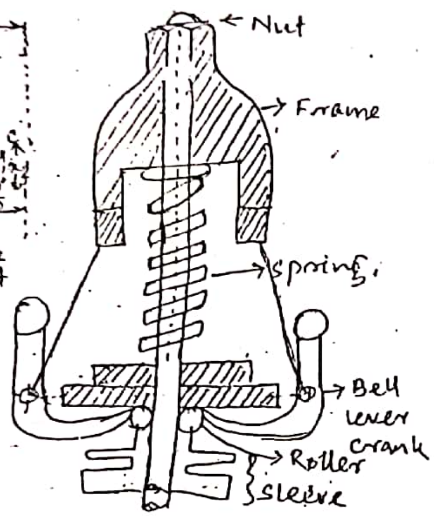
$$\Rightarrow h = \frac{FM}{BM}\left(\frac{m+M}{m}\right) \cdot \frac{895}{N^2}$$

Hartnell Governor–

A Hartnell governor is a spring loaded governor consisting of two bell crank levers pivoted at the point 'O' to the frame of the governor.

Each lever carries a ball at the end of its vertical arm and a roller at the end of its horizontal arm.





A helical spring in compression provides downward force on to two rollers through a collar on the sleeves.

Let m = mass of fly ball (kg)
M = mass of central load on the sleeve (kg)
r = Distance of fulcrum 'O' from the governor axis or the radius of rotation when the governor is having mean speed (m).
r_1 = Min^m radius of rotation (m)
r_2 = Max^m radius of rotation (m)

S_1 = Spring force exerted on the sleeve at when the fly balls are at min^m point (N)
S_2 = spring force exerted on the sleeve at max^m point (N)
ω_1 = Angular speed of governor at min^m point (rad/sec)
ω_2 = Angular speed of governor at max^m point (rad/sec)
x = Length of the vertical arm of the lever (m)
y = length of the ~~spring~~ horizontal arm of the lever (m)
s = stiffness of the spring or the force required to compress or expand the spring by 1mm.
a_1, a_2 = distance of fly ball from 'O' for min^m & max^m respectively
Y_1, Y_2 = distance of the roller from 'O' for min^m & max^m point respectively.
h_1, h_2 = distance of roller from lever axis for min^m & max^m point respectively.

Let the angle of inclination of the horizontal arm to the lever axis θ_1 & θ_2 for min^m & max^m point respectively.

As ~~angle~~ $\angle DOC = \theta_1 + \theta_2$ for min^m & max^m point respectively.

$\therefore \Rightarrow \angle AOB = \theta_1 + \theta_2$ for min^m & max^m point respectively.

In $\triangle AOB$ & $\triangle COD$

$\angle A$ & $\angle D = 90°$

$\angle DOC = \angle AOB = \theta_1$ & θ_2 for min^m & max^m point

So $\angle B = \angle C$.

Hence, $\triangle AOB \cong \triangle COD$ and so

$$\frac{AB}{CD} = \frac{OB}{OC} = \frac{AO}{DO}$$

$\Rightarrow \frac{AB}{BO} = \frac{CD}{OC}$. This is true for case of min^m & max^m point also for min^m point.

$$\frac{a_1}{x} = \frac{h_1}{y}$$

$$\Rightarrow \frac{h_1}{y} = \frac{r - r_1}{x} \quad \dots (1)$$

For max^m point

$$\frac{a_2}{x} = \frac{h_2}{y} \Rightarrow \frac{h_2}{y} = \frac{r_2 - r}{x} \quad \dots (2)$$

Adding eq^n (1) & (2)

$$\frac{h_1 + h_2}{y} = \frac{(r - r_1)(r_2 - r)}{x}$$

$$= \frac{r_2 - r_1}{x}$$

Let $h = h_1 + h_2$

$$\Rightarrow h = (r_2 - r_1)\frac{y}{x} \quad \dots (3)$$

Resolving for min^m point the moment about O;

$$Fc_1 \times x_1 = \frac{W + S_1}{2} y_1 + W . a_1$$

$$\Rightarrow \frac{Mg + S_1}{2} . y_1 = Fc_1 \times x_1 - mg . a_1$$

$$\Rightarrow Mg + S_1 = \frac{2}{y_1}(Fc_1 \times x_1 - mg \times a_1) \quad \dots (4)$$

Similarly taking moment about O for max^m position

$$\frac{Mg + S_2}{2} . y_2 = Fc_2 \times x_2 + mg a_2$$

$$\Rightarrow Mg + S_2 = \frac{2}{y_2}(Fc_2 . x_2 + mg a_2) \quad \dots (5)$$

Substracting equation (4) from eq^n (5)

$$S_2 - S_1 = \frac{2}{y_2}(Fc_2 x_2 + mg a_2) - \frac{2}{y_1}(Fc_1 x_1 - mg a_1)$$

Let $x_1 = x_2 = x$ & neglecting the mass of the fly ball

$\Rightarrow mg = 0$

$y_1 = y_2 = y$

$$S_2 - S_1 = \frac{2}{y}(Fc_2 x) - \frac{2}{y}(Fc_1 x)$$

$$\Rightarrow S_2 - S_1 = \frac{2x}{y}(Fc_2 - Fc_1) \quad \dots (6)$$

Also $S_2 - S_1 = h . s$

$\Rightarrow s = \frac{S_2 - S_1}{h}$ $\quad \because s$ = stiffness of the spring.

Putting the values from eq^n (3) & eq^n (6)

$$s = \frac{(Fc_2 - Fc_1)\, 2x/y}{(r_2 - r_1)\, y/x}$$

$$\Rightarrow s = 2\left(\frac{Fc_2 - Fc_1}{r_2 - r_1}\right)\frac{x^2}{y^2}$$

<u>Sensitiveness of Governor :-</u>

Let there be two governors A & B running at the same speed when the speed increases or decreases by a certain amount the lift sleeves of governor A is greater than the lift of sleeve of B then it is said that the governor A is more sensitive than governor B.

The sensitiveness of the governor is defined as the ratio of difference between max^m & min^m speed to the mean speed.

Let N_1 = Max^m speed (rpm).

N_2 = Min^m speed (rpm)

N = The mean speed $= \frac{N_1 + N_2}{2}$ (rpm)

$$\text{Sensitiveness} = \frac{N_1 - N_2}{N} = \frac{N_1 - N_2}{\frac{N_1 + N_2}{2}}$$

$$= \frac{2(N_1 - N_2)}{N_1 + N_2}$$

If the angular velocity in ω is considered then

Sensitiveness = $\frac{\omega_1 - \omega_2}{\omega}$

where, ω_1 = max^m angular velocity

ω_2 = Min^m angular velocity rad/sec

ω = Mean angular velocity rad/sec.

$\Rightarrow$ Sensitiveness = $\frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$

Stability of Governor –

A governor is said to be stable for every speed with in the working range when there is a definite configuration is there is only one radius of rotation of the governor balls at which the governor is in equilibrium.

For a stable governor if the equilibrium speed increased the radius of governor must also increase.

Isochronus governor –

(A governor is said to be isochronous when the equilibrium speed is constant i.e., the range of speed is zero.) For all radius of rotation of the balls with in the working range neglecting friction.

The isochronism is the stage of ifinite sensitivity.

For the case of porter governor Let N_1 & N_2 be the max^n & min^n speed of rotation in rpm.

$$(N_1)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \cdot \frac{895}{h_1}$$

$$(N_2)^2 = \frac{m + \frac{N}{2}(1+q)}{m} \cdot \frac{895}{h_2}$$

For the isochronism of the governor,

$$N_1 - N_2 = 0$$
$$\Rightarrow N_1 = N_2$$
$$\Rightarrow h_1 = h_2$$

But for the porter governor $h_1 \neq h_2$ so the porter governor can not be isochronous.

Similarly for a ~~iso~~ hartnell governor,

$$Mg + S_1 = 2F_{c1} \frac{x}{y}$$

$$Mg + S_2 = 2F_{c2} \frac{x}{y}$$

$$\Rightarrow Mg + S_1 = 2m r_1 \left(\frac{2\pi N_1}{60}\right)^2 \frac{x}{y}$$

$$\Rightarrow Mg + S_2 = 2m r_2 \left(\frac{2\pi N_1}{60}\right)^2 \frac{x}{y}$$

For isochronisms of the hartnell governor $N_1 = N_2$

$$\frac{Mg + S_1}{Mg + S_2} = \frac{r_1}{r_2}$$

So the isochronism ~~governor~~ of hartnell governor is possible.

Imp

FLY WHEEL –

A fly wheel is used in a machine which serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases energy during the period when the requirement of energy is more than the supply.

The fly wheel controls the speed variation cause by the cyclic fluctuation of energy turning moment by supplying the required energy.

Governor	Flywheel
1. The function of governor is to regulate the supply of driving fluid producing energy according to load requirement so that at different loads almost a constant speed is maintain.	1. The function of fly wheel is to store the available mechanical energy when it is in excess of the load requirement and apart with the same when the available energy is less than that required by the load.
II. It is provided on prime movers like engines and turbines.	II. It is provided on engines and facricating machines like rolling mills punching machine presses etc.
III. It takes care of fluctuation of speed due to variation of load over long range of working of engines and turbines.	III. It takes case of fluctuation of energy during thermodynamic cycle.
IV. Governor works intermitantly i.e., only when there is a change the load.	IV. It works continuously from cycle to cycle.
V. Governor ecomises the consumption of working fluid so that there is no unnecessory consupption.	V. The fly wheel reduces the capital investment and their running expences like in fabrication machine.

Co-efficient of fluctuation of speed—

The difference between the maximum & minimum speed during a cycle is called the maximum fluctuation of speed.

The ratio of maximum fluctuation of speed to the mean speed is called the co-efficient of fluctuation of speed.

Let N_1 & N_2 = The max^m & min^m speed respectively of a cycle.

N = The mean speed of a cycle (rpm).

The co-efficient of fluctuation of speed is given by (C_s)

$$C_s = \frac{N_1 - N_2}{N}$$

$$\text{As } N = \frac{N_1 + N_2}{2}$$

$$\Rightarrow C_s = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

Also $C_s = \frac{2(\omega_1 - \omega)}{\omega_1 + \omega_2}$ (interms of angular velocity)

$C_s = \frac{2(V_1 - V_2)}{V_1 + V_2}$ (in terms of linear velocity).

The reciprocal of the co-efficient of fluctuation of speed is known as the co-efficient of steadyness and is denoted as;

$$m = \frac{1}{C_s} = \frac{N_1 + N_2}{2(N_1 - N_2)}$$

$$= \frac{\omega_1 + \omega_2}{2(\omega_1 - \omega_2)} = \frac{V_1 + V_2}{2(V_1 - V_2)}$$

Fluctuation of energy—

This is the turning moment of a 4-stroke engine.

Let at point A, energy = E

Energy at B = $E - a_1$

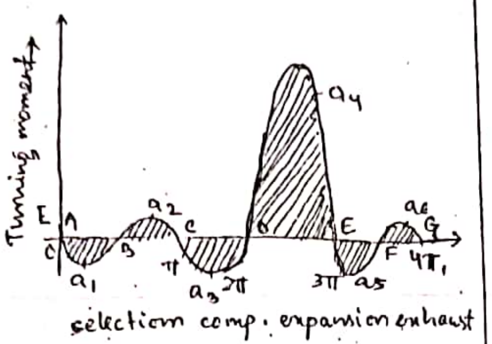
Energy at C = $E - a_1 + a_2$

Energy at D = $E - a_1 + a_2 - a_3$

Energy at E = $E - a_1 + a_2 - a_3 + a_4$

Energy at F

$= E - a_1 + a_2 + a_4 - a_3 - a_5$



Energy at F = $E - a_1 + a_2 - a_3 + a_4 - a_5 + a_6$

Suppose the maximum energy is at B and minimum energies at B.

So the fluctuation of energy = energy at B − ~~energy at E~~ energy at E

$= E - a_1 - E + a_1 - a_2 + a_3 - a_4$

$= a_2 \mp a_3 + a_4$

Energy store in a fly wheel–

For the fly wheel,

Let m = mass of the fly wheel.

k = radius of gyration.

I = Moment of inertia.

N_1 & N_2 = Max^m & min^m speed of fly wheel respectively.

ω_1 & ω_2 = Max^m & min^m angular velocity.

N = Mean speed of fly wheel.

ω = Mean angular velocity of the fly wheel.

C_s = Co-efficient of fluctuation of speed.

The mean kinematic energy of the fly wheel due to its rotation is,

$KE_{mean} = \frac{1}{2} I\omega^2$

$= \frac{1}{2} m(K)^2 \omega^2$

$\Rightarrow KE_{mean} = \frac{1}{2} MK^2\omega^2$

The max^m kinetic energy is,

$KE_{max} = \frac{1}{2} I \omega_1^2$

$KE_{min} = \frac{1}{2} I\omega_2^2$

The maximum fluctuation of energy is given by,

$C_E = KE_{max} - KE_{min}$

$= \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2$

$= \frac{1}{2} I (\omega_1^2 - \omega_2^2)$

$= \frac{1}{2} I \left[(\omega_1 + \omega_2).(\omega_1 - \omega_2)\right]$

$= I . \frac{\omega_1 + \omega_2}{2} \times (\omega_1 - \omega_2)$

$= I \times \omega (\omega_1 - \omega_2)$

Multiplying & dividing ω,

$= I \times \frac{\omega^2}{\omega} (\omega_1 - \omega_2)$

$= I \times \omega^2 \left(\frac{\omega_1 - \omega_2}{\omega}\right)$

$= I . \omega^2 . C_s$

$\Rightarrow C_E = I\omega^2 C_s$

The maximum fluctuation is given by

$I \times \omega^2 \times C_s$

Also as $KE = \frac{1}{2} I\omega^2$

$\Rightarrow 2(KE)_{mean} = C_s$

As the radius of gyration will also be equal to the radius of rim or the fly wheel denoted as r. So the total fluctuation of speed,

$$= 2\times\left[\frac{1}{2}(mr^2\omega^2)\right] C_s \quad \left[\because \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2\right]$$

$$\boxed{\Delta E = mr^2\omega^2 C_s}$$ (In term of angular velocity)

$$= mV^2 C_s$$ (In term of linear velocity)

$$= mr^2\left(\frac{2\pi N}{60}\right)^2 . C_s$$

$$\Delta E = mr^2 N^2 \frac{\pi^2}{900} C_s \text{ (rpm)}$$

$$\boxed{\Delta E = mk^2N^2 . \frac{\pi^2}{900} \times C_s}$$

CHAPTER-5

BALANCING OF MACHINE.

Imp
Dynamometer-

→ Dynamometer is a device by means of which the energy or workdone by a prime mover can be measured. It is a break having additional device to measure frictional resistance by aling the machine to run at the rated speed.

Absorption Dynamometer-

In the absorption dynamometer, the entire energy or power produced by the engine is absorped by the friction resistance of the break and is transformed into heat during the process of measurement. These are further classified as -

1. Prony break dynamometer
2. Rope break dynamometer.

Transmission Dynamometer -

In the transmission dynamometer, the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through the dynamometer to some other machine, where the power devlope is suitable measured.

These are flurthere classified as,

i. Belt transmission
ii. Epicyclic Dynamometer
iii. Torsion Dynamometer.

Prony break dynamometer—

1. It consist of two wooden block placed around a pulley fixed to the shaft of an engine whose

power is required to be measured. The blocks are clamped by means of two bolts & nuts.

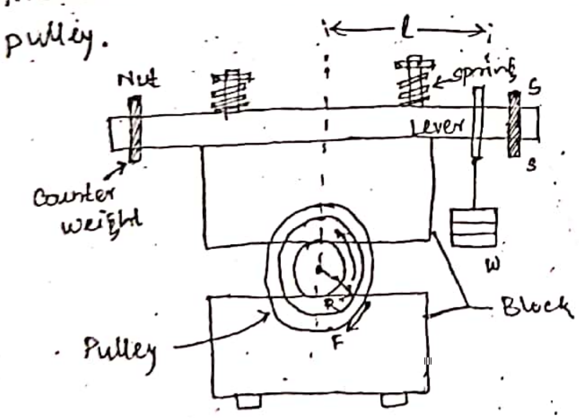
ii. A helical spring is provided between the nut & the upper block to adjust the pressure on the pulley to control its speed.

iii. The upper block has a long lever attached to it and carries a weight 'W' at its outer end.

iv. A counter weight is placed at the other end of the lever which balances the break when unloaded.

v. Two steps S, S are provided to limit the motion of the lever. When the break is to be put in the operation, the long end of the lever is loaded with suitable weight 'W' and the nuts are tightened untill the engine shaft run at a constant speed and the lever is in horizontal position.

Under these conditions, the moment due to weight 'W' must balance the moment of the frictional resistance between the block & the pulley.



Let, W = weight at the outer end of the lever in (N)

L = Distance betn centre of pulley to the weight

F = Frictional resistance betn the block & the pulley (N)

R = Radius of the pulley (m)

N = Speed of the shaft (rpm)

Moment of the frictional resistance or torque to the shaft,

$$T = W \times L = F \times R$$

Workdone in one revolution = Torque × Angle turned in radian

$$= T \times 2\pi$$

We know the break power of the engine,

$$B.P. = \frac{\text{Workdone per min}}{60}$$

$$= \frac{T \times 2\pi N}{60} \text{ Watts.}$$

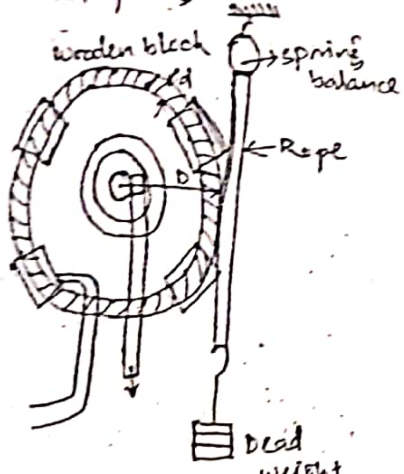
Rope brake dynamometer –

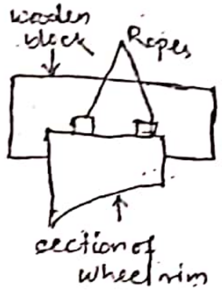
1. Rope brake dynamometer is most commonly used for measuring the break power of the engine. It consists of one, two or more ropes wound arround the fly wheel or rim of the pulley fixed or rigidly to the shaft of an engine.

ii. The upper end of the rope is attached to a spring balanced while the lower end of the rope is attached to a spring balanced while the lower end of the rope is kept in position by applying a dead weight as shown in figure.

iii. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals arround the circumference of the fly wheel.

iv. In the operation of the break, the engine is made to run at a constant speed. The frictional torque due to the rope must be equal to the torque being transmitted by engine.





Let W = Dead load (N)

S = spring balance reading (N)

D = Diameter of the wheel (M)

d = Diameter of the rope (m)

N = speed of the engine shaft (rpm)

Net load on the brake $= (W-S)$

Distance moves in one revolution $= \pi(D+d)$ mtr.

Workdone per revolution $= (W-S)\pi(D+d)$ N-m;

Workdone per minute $= (W-S)\pi(D+d)\times N$ N-m.

Break power of the engine–

$$B.P. = \frac{\text{Workdone per min}}{60}$$

$$= \frac{(W-S)\pi(D+d)\times N}{60} \text{ watt}$$

If the diameter of the rope d is neglected,

then, $B.P. = \frac{(W-S)\pi DN}{60}$ watts.

Crowing pulleys–

1. The rim of the pulley of a flat belt drive is slightly crowned to prevent the slipping off belt from the pulley.

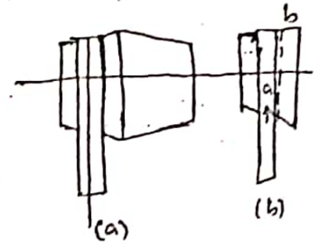
ii. The crowing can be in the form of conical surface or a convex surface.

iii. Assume that some how a belt ab comes over the conical portion of the pulley and takes the position as shown in figure.

iv. Its centre line remains a plane, the belt will touch the rim surface at one edge only.

v. This impractically owing to the pull, the belt always tends to stick to the rim surface.

vi. This belt is also a lateral stiffness; Thus a belt has to bend in the way shown in figure.



a. Let the belt travel in the direction of arrow, as the bend touches the cone, the point 'a' on it tends to adhere to cone surface due to pull on the belt. This means as the pulley will turn a quarter turn, the point on the belt will be carried to 'b' which is towards the mid-plane of the pulley than that previously occupied by the edge of the belt.

But again the belt can not be stable on the pulley in the upright position and has to bend to stick to the cone surface. It will occupy the position shown by dotted lines. Thus if a pulley is made up of two equal cones or convex surface, the belt will tent to climb on the slopes and will thus run with its centre line on the mid plane of the pulley. The ~~normal~~ amount of crowing is usually 1/96 of the pulley face width.

CHAPTER-6 VIBRATION OF MACHINE PARTS.

Amplitude –
It is defined as its maximum displacement of a vibrating body from its equillibrium body.

Time Period –
It is the time taken by the motion to repeat itself and is measured in second.

Imp. Frequency –
Frequency is the no. of cycle of motion completed in one second. It is expressed in hertz(Hz) and is equal to cycle per second. Reciprocal of time period.

Imp. Free vibration –
Free or natural vibration are the elastic vibration in which there is no friction and external force after the initial release of the body.

Imp. Damped vibration –
Damped vibration are the vibration when the energy of vibrating system is gradually dissipated by friction and other resistances. The variation gradually cease and the system rest in its equillibrium position.

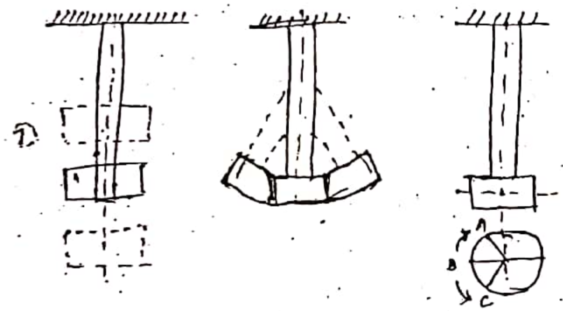
Imp. Forced vibration –
Forced vibration occurs when repeated force contineous acts on a system. The frequency of the vibration is that of the applied force

and is independent of their own natural frequency.

5. Types of vibration—

Consider a vibrating body ex- a rod, shaft or spring figure shows a mass less shaft. One end of which is fixed and the other end carry a heavy disc.

1. Longitudinal vibration
2. Transverse vibration
3. Torsional vibration.



Imp. 1. Longitudinal vibration—

'If the shaft of elongated and shortend so that the same move up and down resulting in tensile and compressive stress' in the shaft to vibration are said to be longitudinal. The different particles of the body move parallel to the axis of the body.

Imp. 2. Transverse vibration—

When the shaft is bent alternatively and tensile and compressive stress due to bending result, the vibration are said to be transverse. The particles of the body move approximately perpendicular to its axis.

3. Torsional vibration—

When the shaft is twisted & untwisted alternatively & torsional shear stress are induced, the vibrations are known as torsional vibration. The particle of body move in a circle about the axis of shaft.

Natural frequency or free longitudinal vibration—

The natural frequency may be found by following method.

1. Equillibrium Method—

I. It is based on the principle that when ever a vibratory system is in equillibrium the algebric sum of forces and moments acting on it is zero.

II. This is in accordance with D'almbert's principle that the sum of inertia force and the external forces on a body in equillibrium must be zero.

III. Consider a constraint of negligible mass in an unstrained position as shown in figure below.

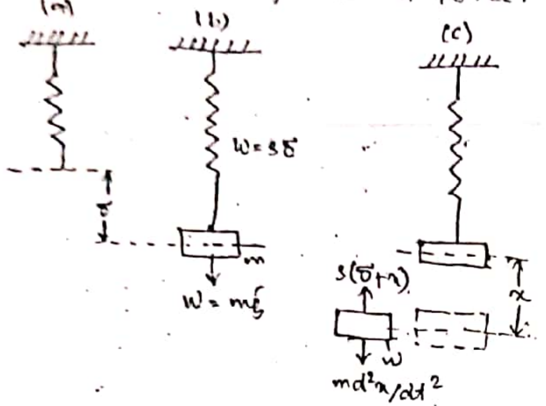
Let, S = stiffness of the constraint or the force required to produce unit displacement in the direction of vibration expressed in N/m.

m = Mass of the body (kg)

W = weight of the body (N) = m.g

δ = static deflection of the spring (m)

x = Displacement by external force.



In the equilibrium position as shown in fig(b), the gravitational pull W = mg is balanced by a force of spring such that W = s.δ.

But in fig-(c), since the mass is now displaced from its equilibrium position by a distance x and is than released. Therefore after time t.

Restoring force = $W - s(\delta + x)$

$= W - s\delta - sx$

$= s\delta - s\delta - sx$

$= -sx \quad \dots ①$ (Taking upward force as −ve)

Accelerating force = mass × acceleration

$= m \times \frac{d^2x}{dt^2} \quad \dots ②$ (Taking downward force as +ve)

Equating eqⁿ − ① & ② we get,

$m \cdot \frac{d^2x}{dt^2} = -sx$ or $m \cdot \frac{d^2x}{dt^2} + sx = 0$

$\therefore \frac{d^2x}{dt^2} + \frac{s}{m}x = 0 \quad \dots ③$

We know that the fundamental equation of simple harmonic motion,

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \dots ④$$

Equating ③ & ④ we get,

$$\omega = \sqrt{\frac{s}{m}}$$

Time period −

$$t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$$

Natural frequency is given by,

$$f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \quad (\because mg = s\delta)$$

$$\rightarrow \boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ Hz.}}$$

2. Energy Method −

I. The kinetic energy is due to the motion of the body and potential energy is w.r.t. a certain datum position. which is equal to the amount of work required to move the body from the datum position.

II. In case of vibration the datum position is the mean or vibration position at which potential energy of the body is zero.

III. In the free vibration, no energy is transfered to the system from the system.

IV. The kinetic energy & potential energy must be a constant quantity which is same at all the times.

$$\therefore \frac{d}{dt}(KE + PE) = 0 \quad \dots ①$$

$KE = \frac{1}{2} \times m \left(\frac{dx}{dt}\right)^2 \ldots (2)$

$PE = \left(\frac{0+sx}{2}\right)x = \frac{1}{2} sx^2 \ldots (3) \Rightarrow$ ($P.E =$ mean force $\times$ displacement)

So, $\frac{d}{dt}\left[\frac{1}{2}\times m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2} s.x^2\right] = 0$

$\Rightarrow \frac{1}{2} m \times 2 \times \frac{dx}{dt} \times \frac{d^2x}{dt^2} + \frac{1}{2} \times s \times 2x \times \frac{dx}{dt} = 0$

$\Rightarrow m \times \frac{d^2x}{dt^2} + s.x = 0$ or $\frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0$

v. The time period and natural frequency may be obtained as discussed in previous method.

3. Ray leigh's Method—

i. In this method, the maximum KE at mean position is equal to the maximum PE at the extream position.

ii. The time period & natural frequency may be obtained as discussed in the previous method.

$x = X \sin \omega t \ldots (1)$

Where, $x =$ Displacement of body from the mean position after t second.

$X =$ Max^m displacement from mean position to extream position.

$\frac{dx}{dt} = \omega \times X \cos \omega t.$

Since at the mean position $t = 0$. Therefore maximum velocity at the mean position,

$v = \frac{dx}{dt} = \omega X$

$\therefore$ Max^m KE at mean position,

$= \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 X^2 \ldots (2)$

Max^m PE at extream position,

$= \left(\frac{0+sX}{2}\right)X = \frac{1}{2} s.X^2 \ldots (3)$

($\because$ PE = mean force $\times$ displacement)

Equating (2) & (3),

$\frac{1}{2} \times m\omega^2 X^2 = \frac{1}{2} \times sX^2$

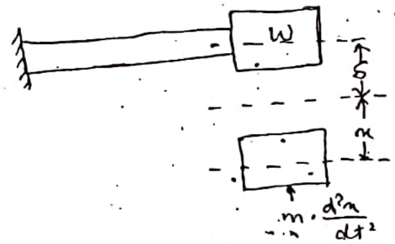
or $\omega^2 = \frac{s}{m}$ & $\omega = \sqrt{\frac{s}{m}}$

$\therefore$ Time period—

$t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{s}{m}}$

Natural freqnce of free transverse vibration—

Consider a shaft of negligible mass, whose one end is fixed and the other end carries a body of weight w.



Let $s =$ stiffness of shaft.

$\delta =$ static deflection due to weight of body.

$x =$ Displacement of body from mean position.

$m =$ Mass of body $= w/g$

Restoring force $= -sx$

Accelerating force $= m \times \frac{d^2x}{dt^2}$

$m \times \frac{d^2x}{dt^2} = -sx$

$\Rightarrow m \times \frac{d^2x}{dt^2} + sx = 0$

$\therefore \frac{d^2x}{dt^2} + \frac{s}{m} x = 0$

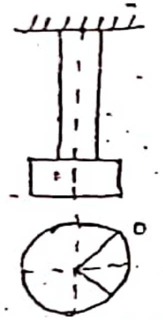
Hence the time period & natural frequency of the transverse vibration are same as that of longitudinal vibration. Therefore,

Time period, $t_p = 2\pi\sqrt{\frac{m}{s}}$

Natural frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

Free torsional vibration–

Consider a shaft of negligible mass whose one end is fixed and other end is carrying a disc. If the disc is given a twist about its vertical axis then released it will starts ~~conti~~ oscillatory about the axis and will perform torsional vibration.



Let θ = Angular displacement of the shaft

m = Mass of disc (kg)

I = Mass of moment of inertia of disc in $kg\,m^2 = mk^2$

K = Radius of gyration (m)

q = Torsional stiffness (N-m)

Restoring force $= -q\theta$ .... ①

Accelerating force $= I \times \frac{d^2\theta}{dt^2}$ .... ②

Equating 1 & 2

$I \times \frac{d^2\theta}{dt^2} = -q\theta$

$\Rightarrow I \times \frac{d^2\theta}{dt^2} + q\theta = 0$

$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{q}{I}\theta = 0$ .... ③

The fundamental eqn of simple harmonic motion is, $\frac{d^2\theta}{dt^2} + \omega^2 x = 0$ .... ④

Comparing ③ & ④

$\boxed{\omega = \sqrt{\frac{q}{I}}}$

Time period $(t_p) = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{q}}$

Natural frequency $(f_n) = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$

The value of torsional stiffness q may be obtained from the torsion equation.

$\frac{T}{J} = \frac{C\theta}{l}$ or $\frac{T}{\theta} = \frac{CJ}{l}$

$q = \frac{CJ}{l}$ $\quad (\because \frac{T}{\theta} = q)$

Where, C = Modulus of rigidity for the shaft material.

J = Polar moment of inertia of the shaft cross-section

$= \frac{\pi}{32} d^4$

Where,

d = diameter of shaft

l = length of shaft.

Imp.

Causes of vibration–

1. Un balanced reciprocating parts.
2. Unbalanced rotating machine parts.
3. Incorrect alignment of the transmission element such as coupling etc.
4. Use of simple spur gears for power transmission
5. Worn-out teeth of gear of the power transmission.
6. Impact taking parts of their machine explosion.
7. of impact of working fluid of prime movers.
7. Loose transmission belt & chain.
8. Loose fastening of the moving parts..
9. Vibration waves from other sources and machine installed nearly due to improper isolation of vibration from them.
10. Due to more material contact such as bases plate on the foundation for the pedestal bearing.
11. Non-rigid machine foundation due to ~~too~~ lack of compact soil below causing settelement of machine component.

Remedies of vibration–

Although it is impossible to eliminate the vibration yet these can be reduced by adopting various remedies.

1. Partial balancing of reciprocating masses.
2. Balancing of unbalanced rotating masses.

III. Using helical gear instead of spur gases.
IV. Proper tightening and locking of fastening & periodically ensuring it again.
V. Correcting the mis-alignment of rotating component and checking it from time to time.
VI. Timely replacement of workout moving parts, slides and bearing with excessive clearance.
VII. Isolating vibration from other machine & source by providing vibration insulation pads in the machine foundation.
VIII. Making machine foundation on compact ground & making them sufficiently strong so that they do not yeild or settle under the load of the machine.

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